Digitizers and dynamic range

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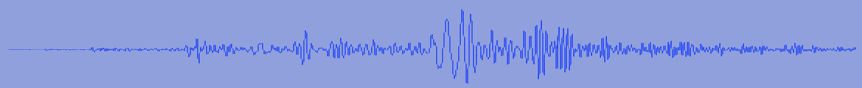
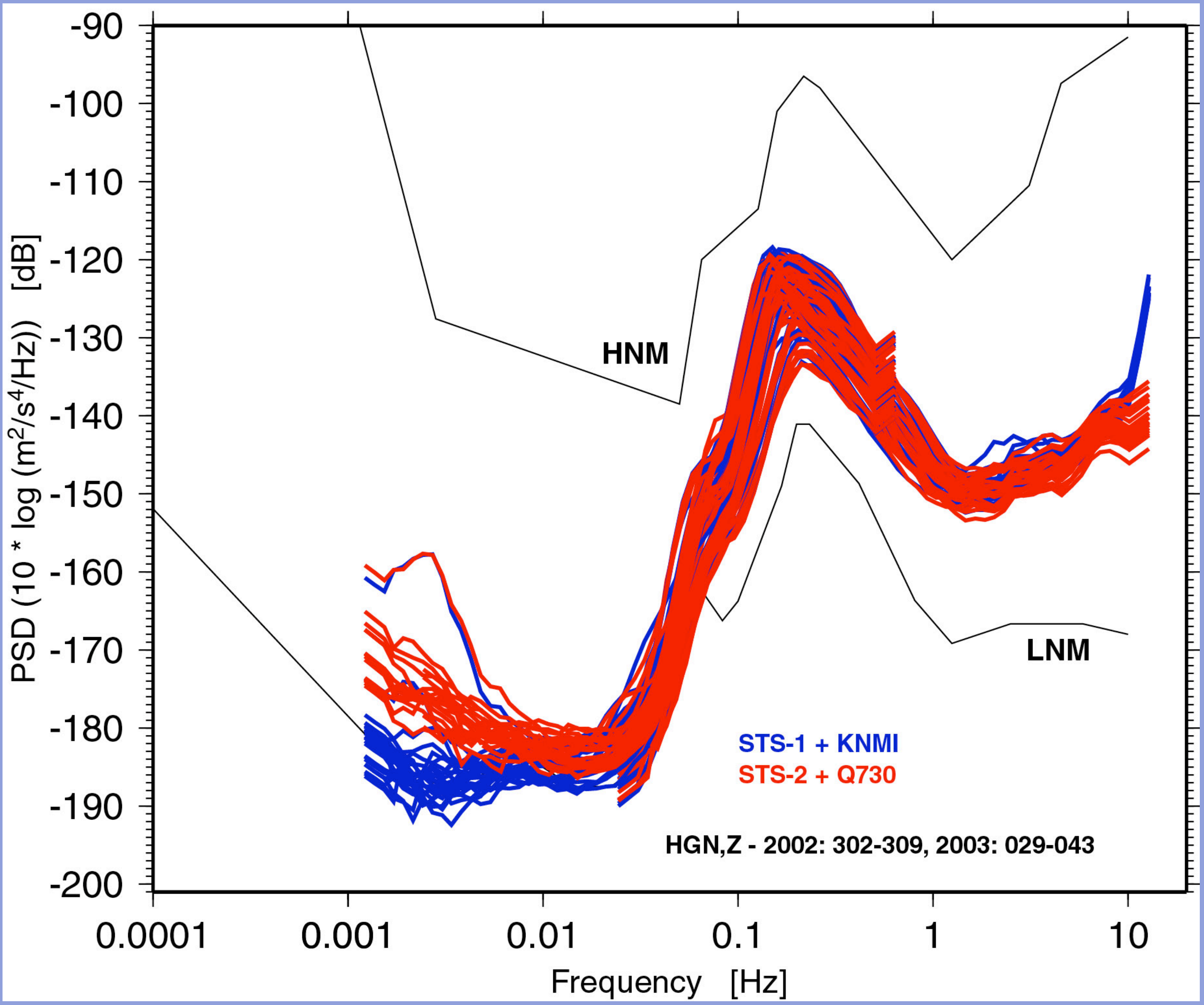


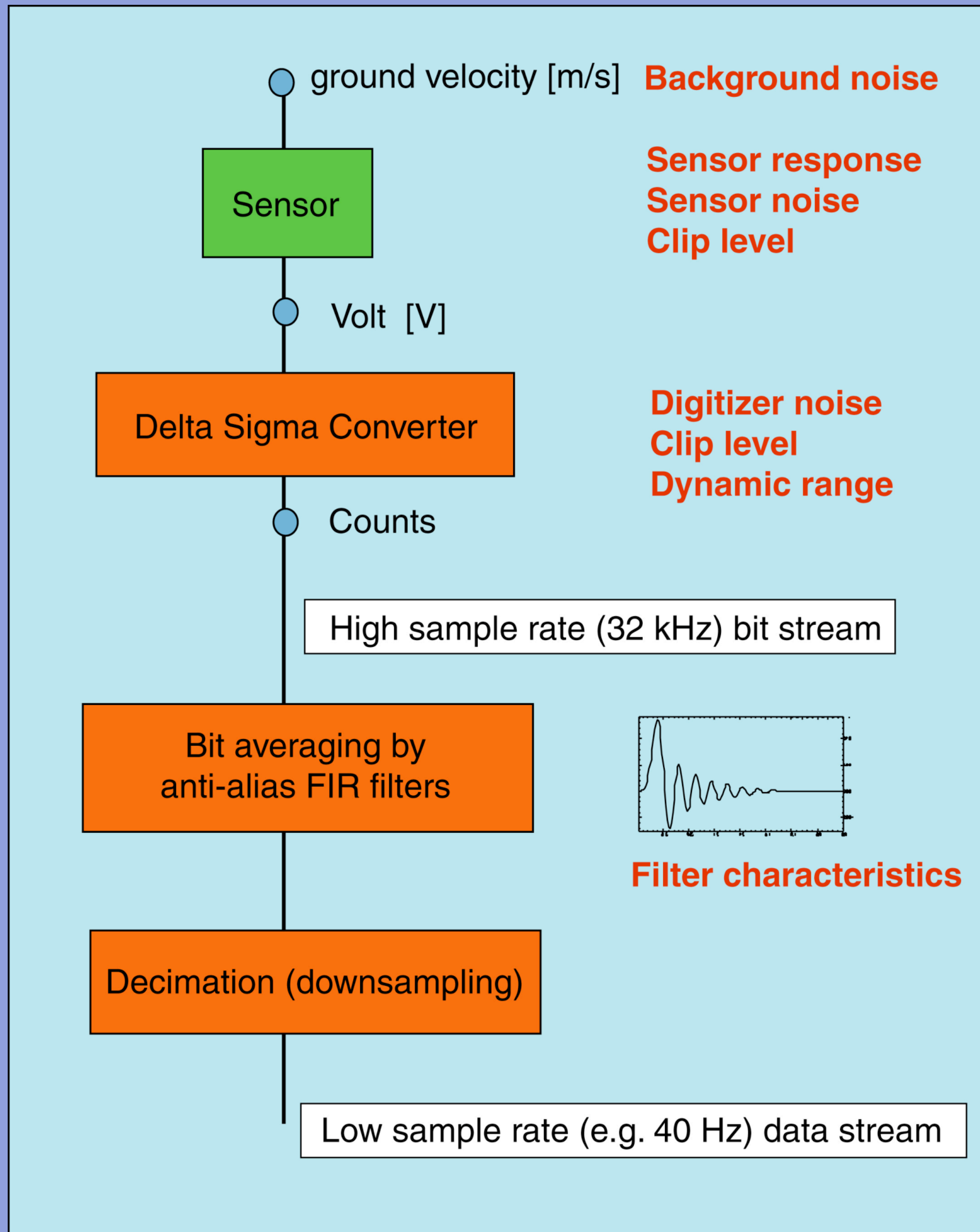
IRIS / ORFEUS Workshop
Understanding and Managing
Information from Seismological Networks

28 Feb – 4 Mar 2005, Palmanova, Italy

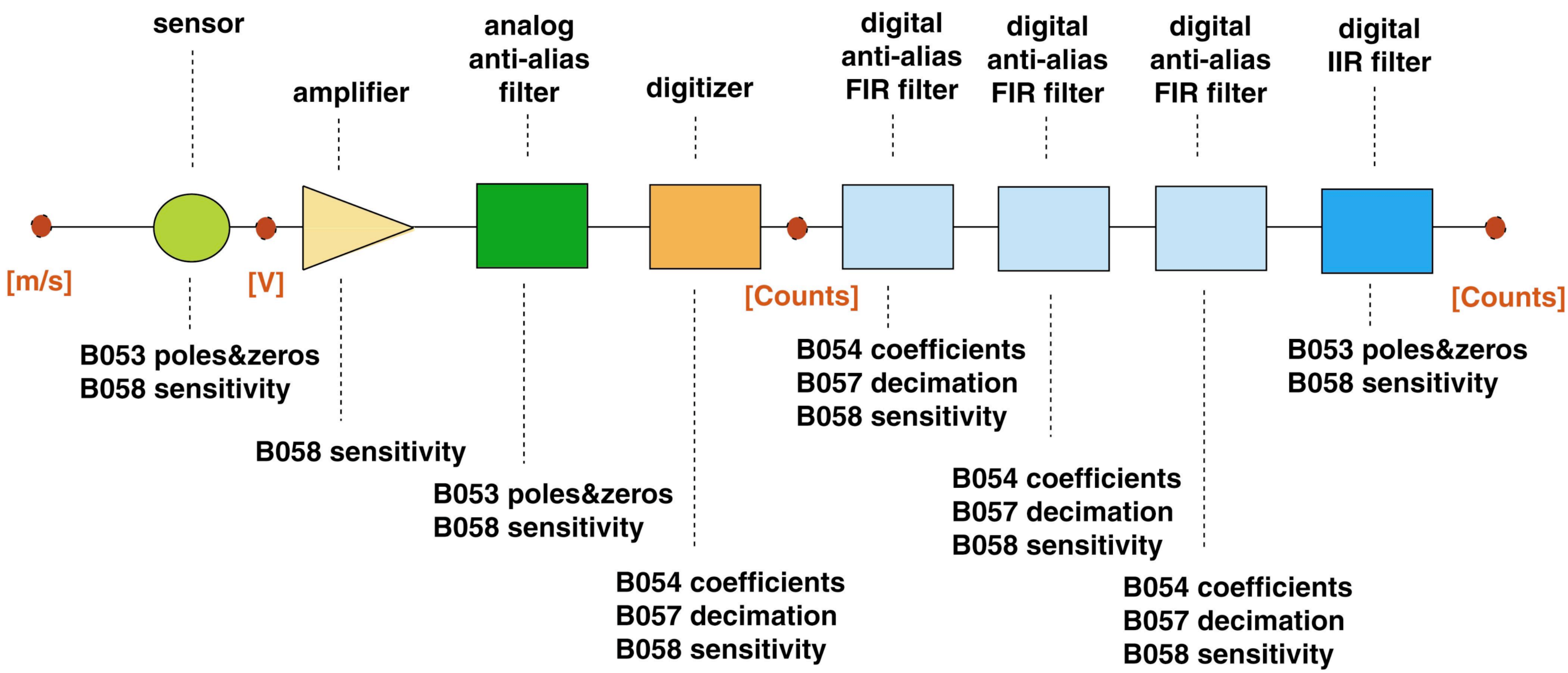


PSD (10 * log (m2/s4/Hz)) [dB]
-90
-100
-110
-120
-130
-140
-150
-160
-170
-180
-190
-200
0.0001
0.001
0.01
0.1
1
10
Frequency [Hz]
HNM
LNM
STS-1 + KNMI
STS-2 + Q730
HGN,Z - 2002: 302-309, 2003: 029-043

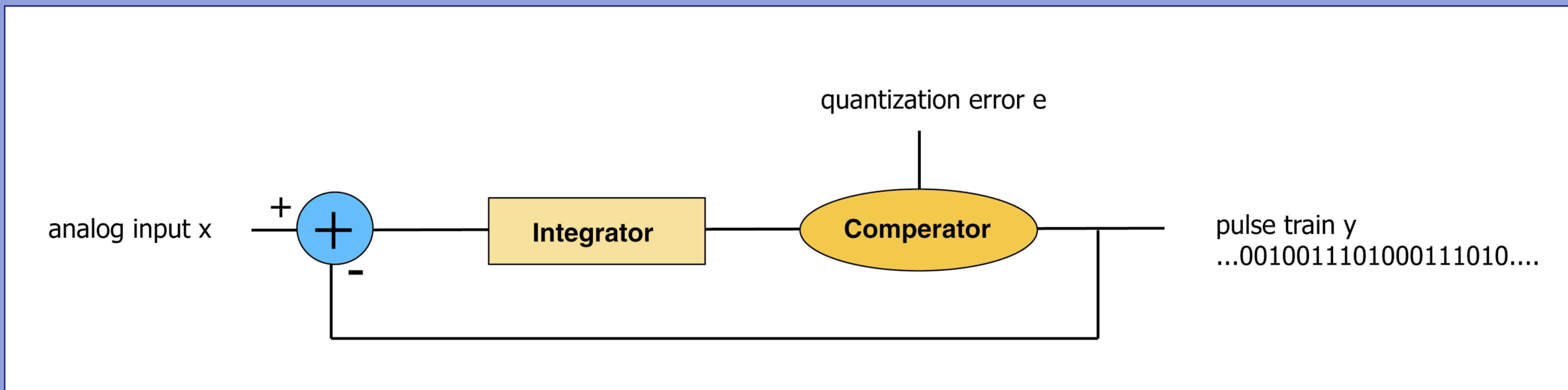
Seismograph system



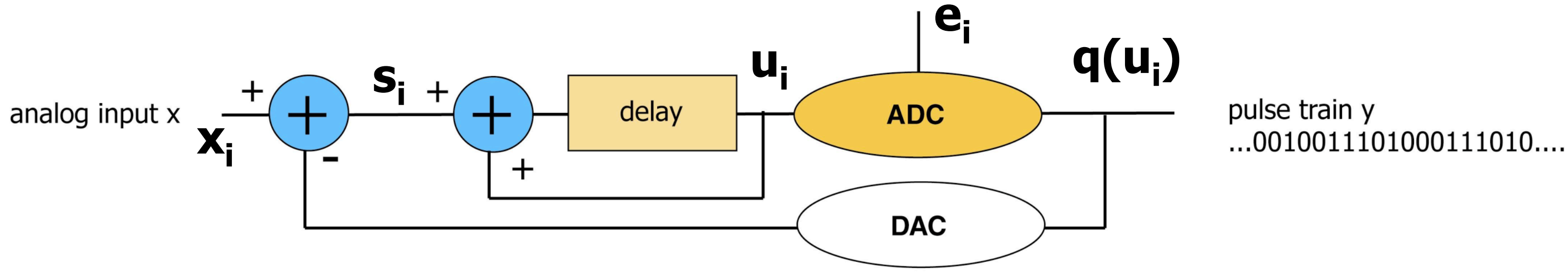
Acquisition system block diagram and SEED response blockettes



Oversampled Delta-Sigma A/D Digitizer
(one-bit noise shaping converter)



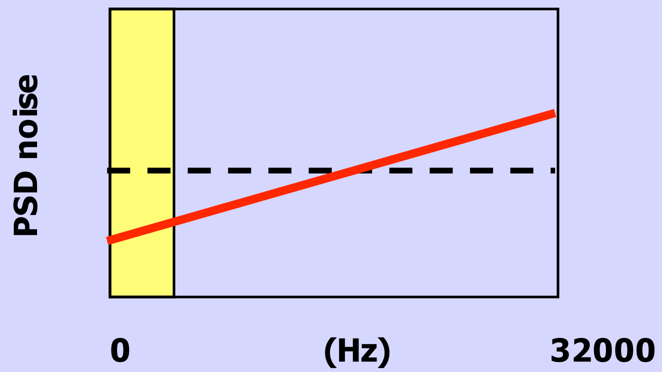
Comperator: ADC or quantizer
Feedback: average of y follows the average of x
Integrator: accumulates the quantization error e over time
Pulse train: pulse density representation of x
Oversampling: increase of resolution



$$
\left.\begin{aligned}
s_i &= x_i - q(u_i) \\
u_i &= s_{i-1} + u_{i-1} \\
q(u_i) &= u_i + e_i
\end{aligned}\right\} \quad q(u_i) = x_{i-1} + (\underbrace{e_i - e_{i-1}})
$$

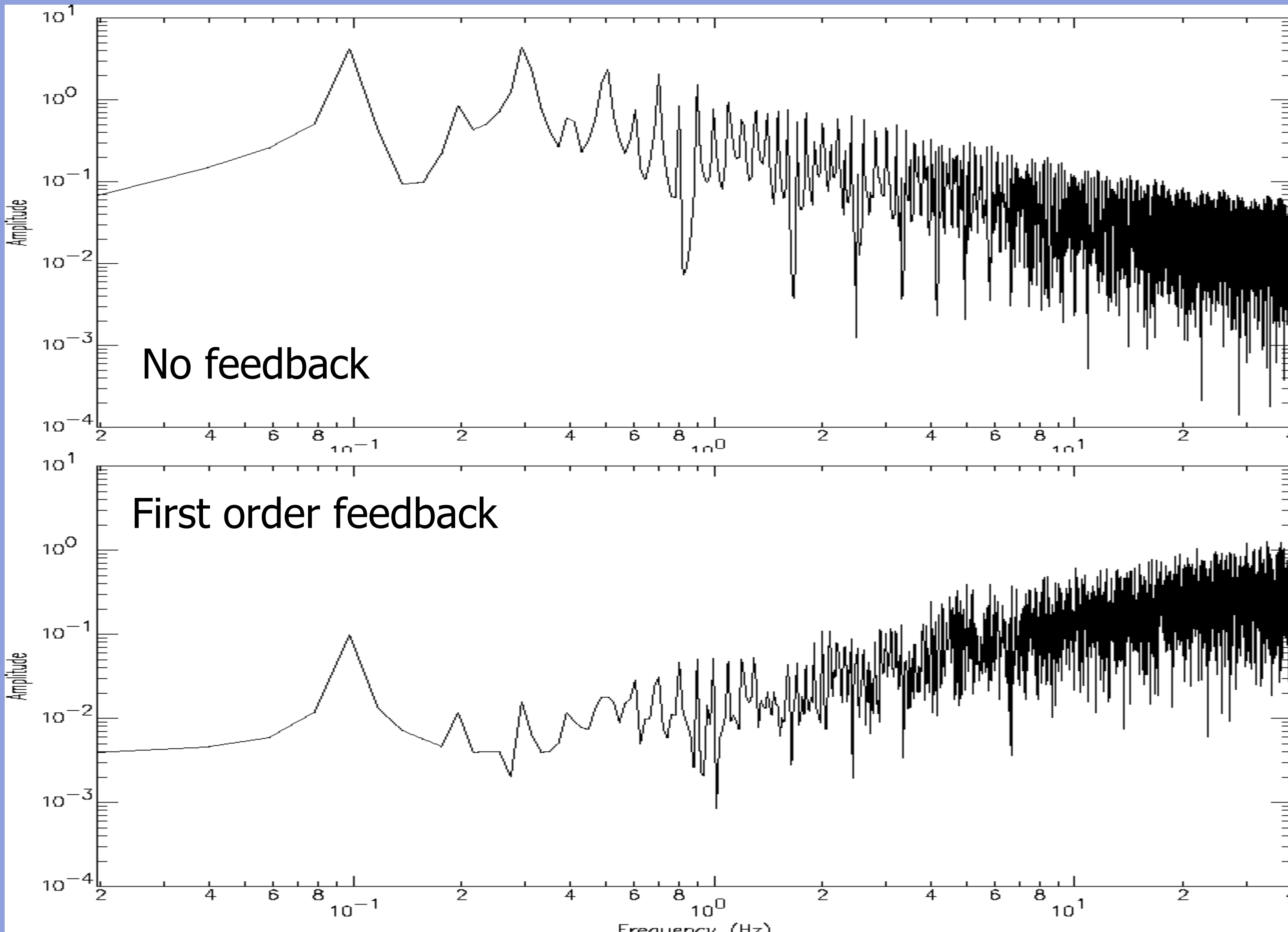
Noise shaping

2-nd order: $$q(u_i) = x_{i-1} + \overbrace{(e_i - 2e_{i-1} + e_{i-2})}$$



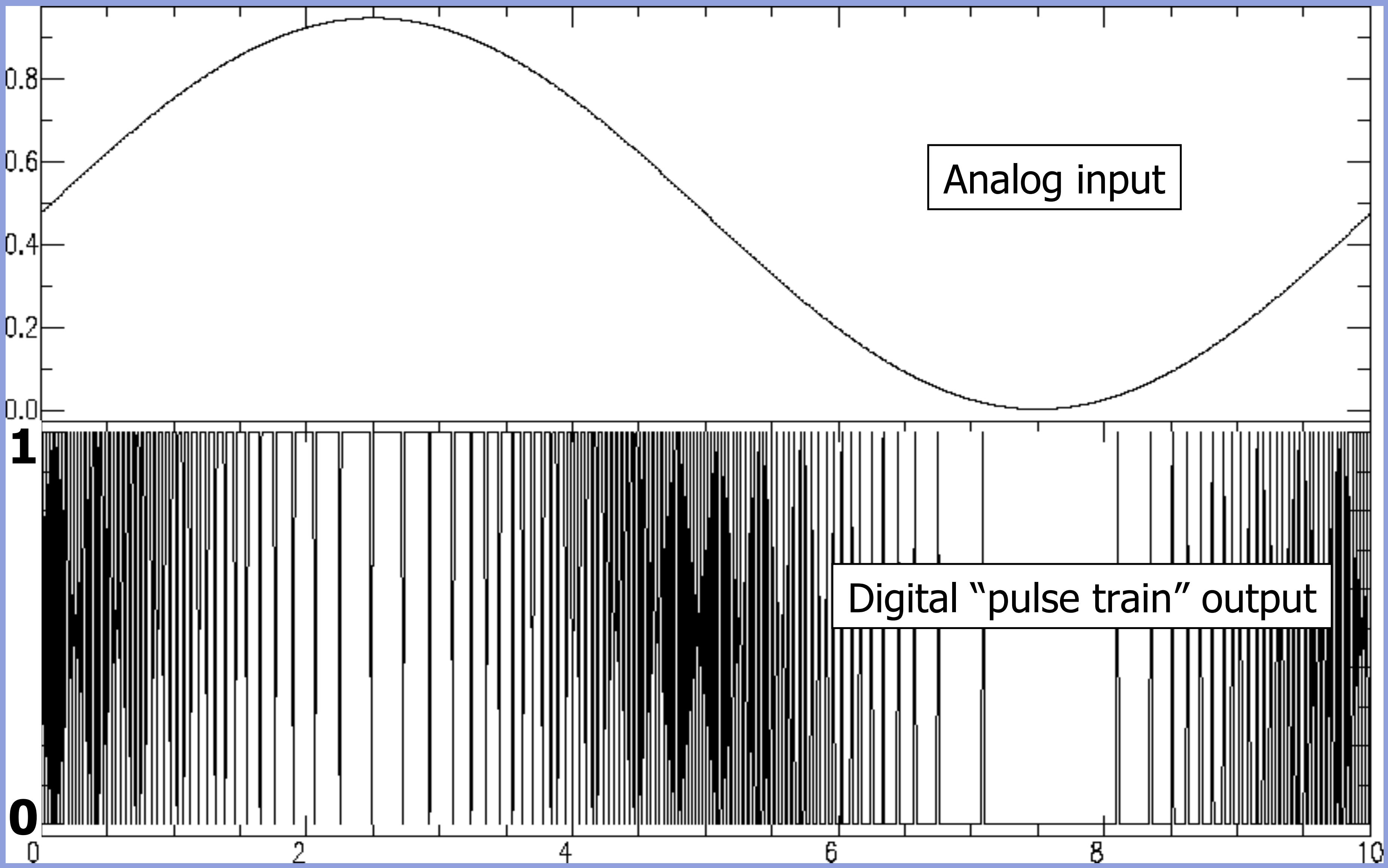
Assumption: quantization noise is white noise

1-bit ADC: quantization error spectrum (oversampling factor 2)

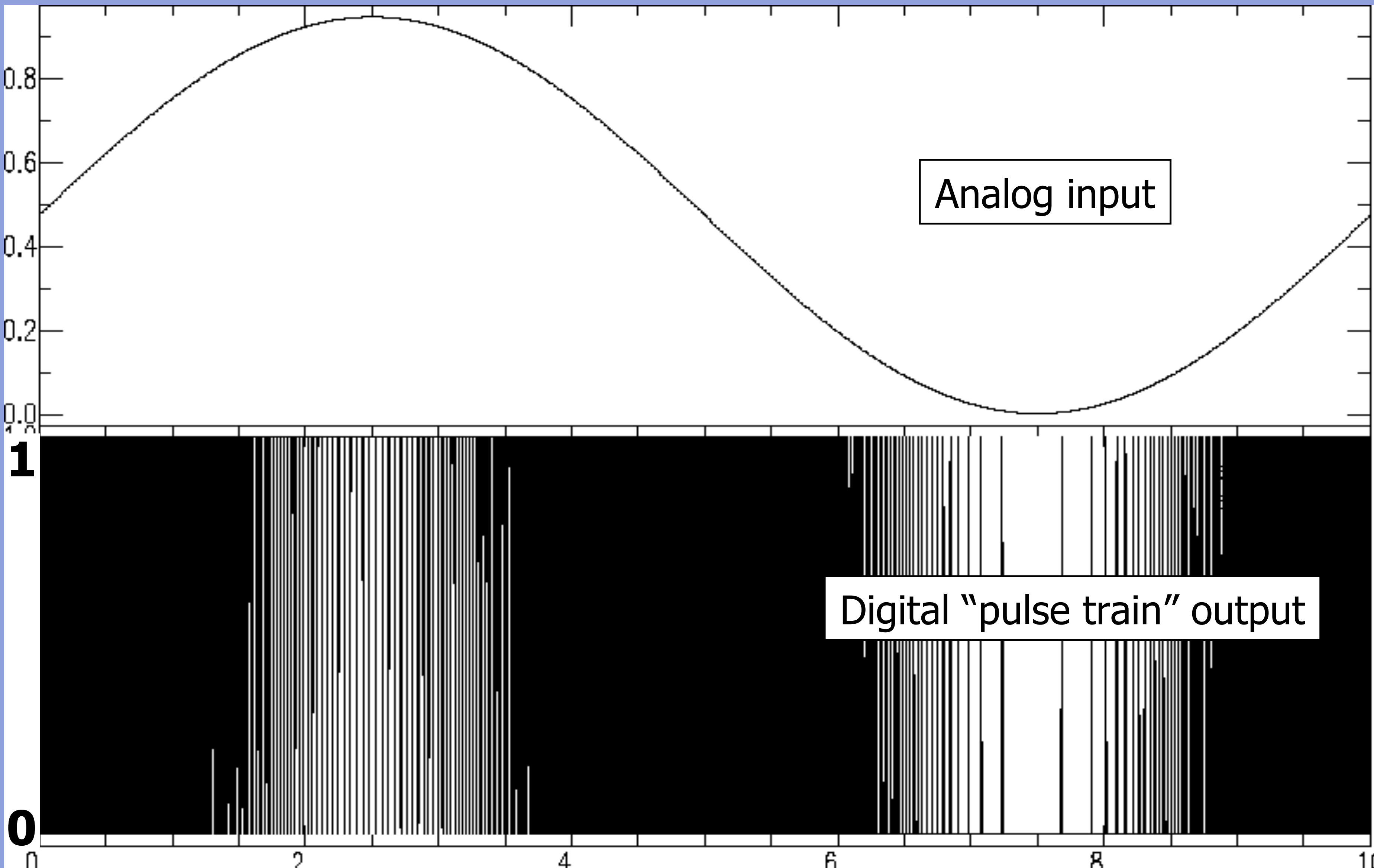


demo

Oversampling factor: 2



Oversampling factor: 5

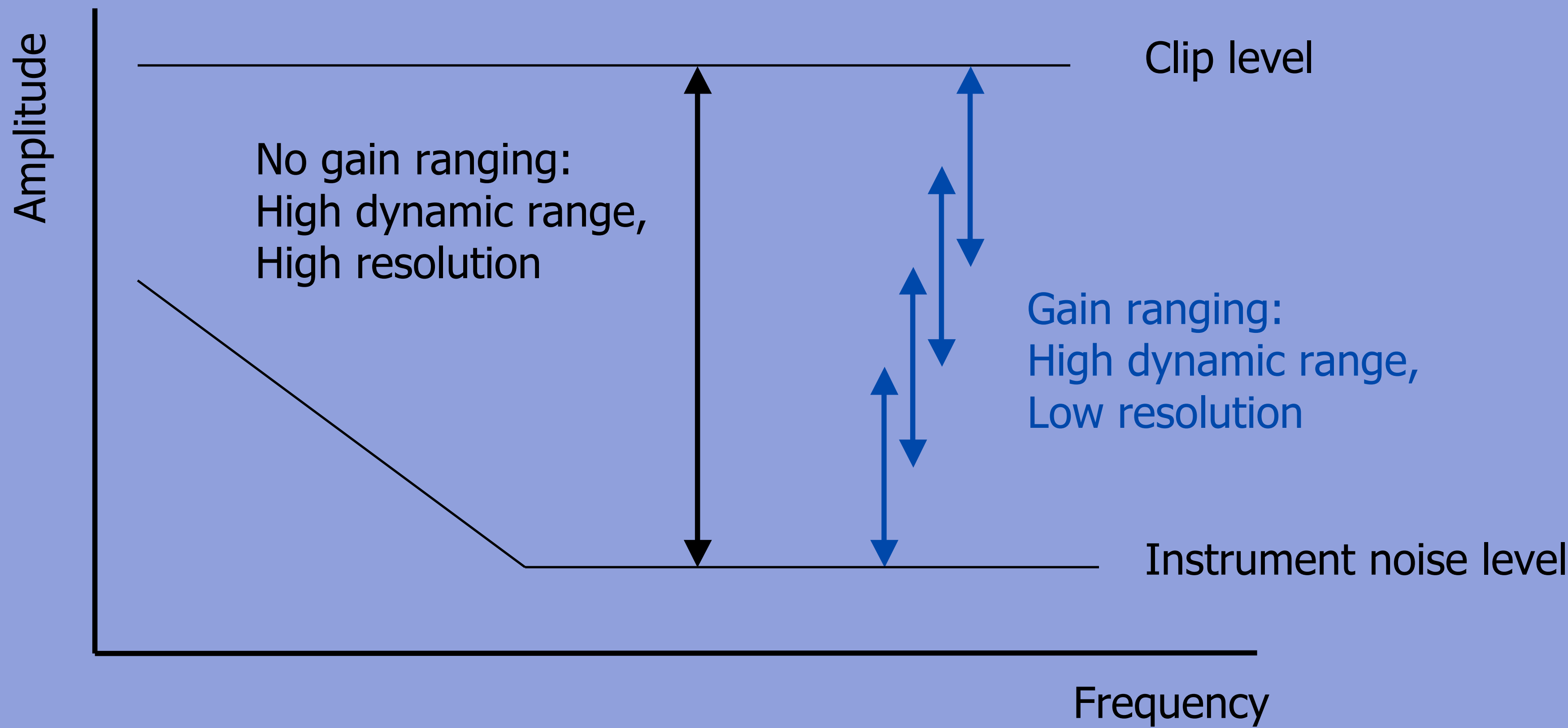


Very high sampling rate ADC with very poor resolution (1 bit)

- Feedback ⟹ quantization error reduction

What is the relation with
“**dynamic range**”
and
“**resolution**”
?

Dynamic range / resolution



Dynamic range is frequency dependent

Dynamic range of a digitizer – some theory:

Quantization:

$$e = q(x) - x$$

Variance of error:

$$e_{rms}^2 = \int_{-\infty}^{\infty} \left[q(x) - x \right]^2 p(e) de = \frac{1}{\Delta} \cdot \int_{-\Delta/2}^{\Delta/2} e^2 de = \frac{\Delta^2}{12}$$

Number of quantization levels:

$$\frac{2A}{\Delta} = 2^n$$

Dynamic range:
(Benett, 1948)

$$SNR = 10 \cdot \log\left(\frac{s_{rms}^2}{e_{rms}^2} \right)$$

$$s_{rms}^2 = A^2 / 2$$

$$\Rightarrow \quad SNR = 10 \cdot \log\left(\frac{A^2 / 2}{\Delta^2 / 12} \right) = 1.76 + n \cdot 6.02$$

↓ 6 dB per bit

$$SNR(f) = 10 \cdot \log\left(\frac{PSD_{\max}(f)}{PSD_{\min}(f)}\right)$$

$$PSD_{\min} = 10 \cdot \log\left(\frac{\Delta^2}{12} \cdot \frac{1}{f_{Nyquist}}\right)$$

$$PSD_{\max} = 10 \cdot \log\left(\frac{A^2}{2} \cdot \frac{1}{f_{Nyquist}}\right)$$

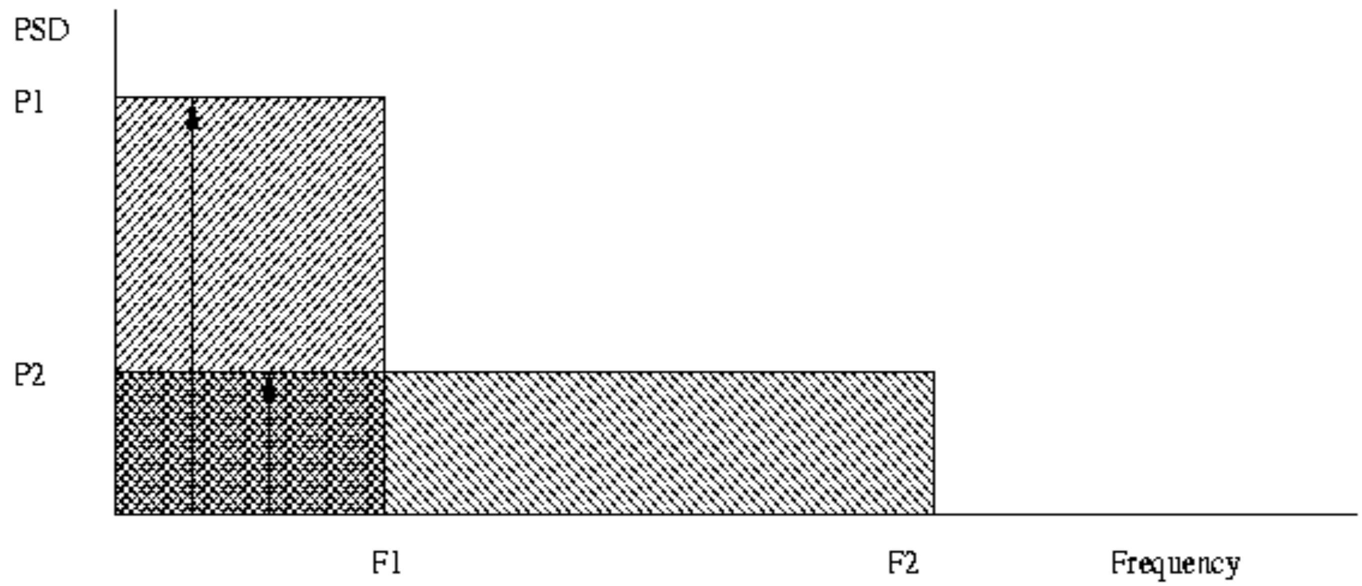
$$SNR = 10 \cdot \log\left[\frac{V_{pp}{}^2}{2} \cdot \frac{1}{f_{Nyquist}}\right] - 10 \cdot \log\left[\left(\frac{V_{pp}}{2^n}\right)^2 \cdot \frac{1}{12 \cdot f_{Nyquist}}\right]$$

rel. to 1 V^2/Hz

V_{pp}: peak-to-peak input in V

n: number of bits

F_{nyt}: Nyquist frequency


PSD
P1
P2
F1
F2
Frequency

Very high sampling rate ADC with very poor resolution (1 bit)

- Feedback ⟹ quantization error reduction
- Oversampling ⟹ higher resolution
- Bitstream averaging + decimation ⟹ low sample rate

⇓

Improved dynamic range and high resolution at a lower effective sampling rate

Demonstration

Java application of a delta-sigma modulator

How to measure the dynamic range of a datalogger

- **shorten the input and the record self noise**
  - ratio of maximum peak amplitude and clip level
  - specify frequency band:

    (RMS noise) / (RMS full scale sine)

  - PSD graph, as function of the frequency
- **common input signal**
  - PSD graph (cross spectral analysis)

Dynamic range = (RMS noise) / (RMS full scale sine)

Shortened inputs:

Q4120: $V_{pp} = 40$ V $\Rightarrow$ $V_{rms} = 14.1$ V

dt = 0.05 s

0.01 – 8 Hz: RMS noise 0.8 uV (measured)

} 143.8 dB

rel to 1 V^2 / Hz

at 20 sps

Dynamic range = (RMS noise) / (RMS full scale sine)

Shortened inputs:

Q4120: V_{pp} = 40 V ⇒ V_{rms} = 14.1 V

dt = 0.05 s

0.01 – 8 Hz: RMS noise 0.8 uV (measured)

143.8 dB

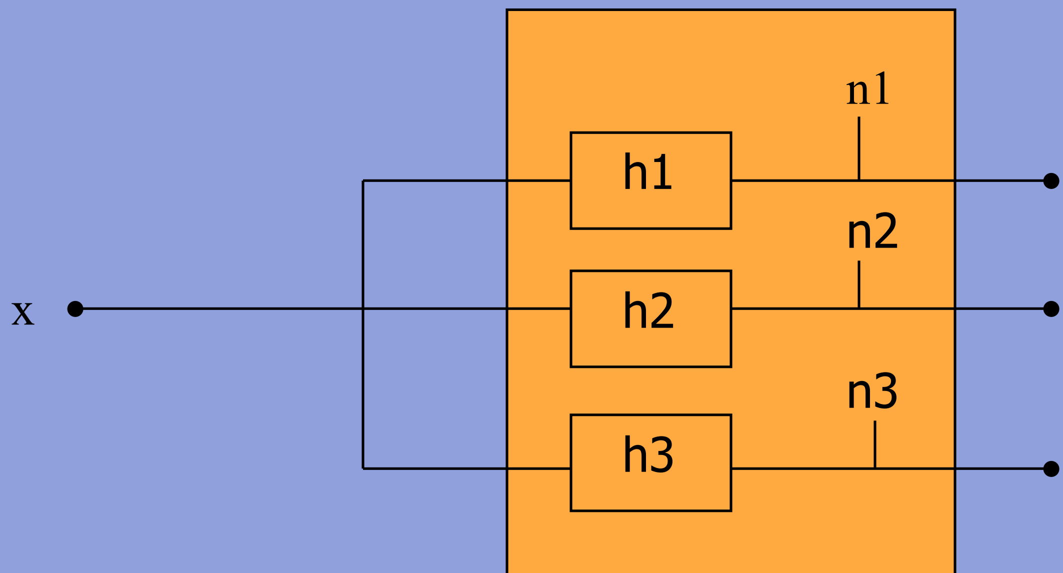
rel to 1 V^2 / Hz

at 20 sps

$$SNR = 10 \cdot \log\left(\frac{A^2/2}{\Delta^2/12}\right) = 1.76 + n \cdot 6.02$$

→ 23.7 bits

Cross spectral analysis
Linear noise-model



y1 = x ⊗ h1 + n1

y2 = x ⊗ h2 + n2

y3 = x ⊗ h3 + n3

Assumptions

(1) $\int x \cdot n1 = \int x \cdot n2 = \int x \cdot n3 = 0$

(2) n1, n2 and n3 are uncorrelated

Auto/Cross correlation

Time domain

$y1 = x \otimes h1 + n1$

$y2 = x \otimes h2 + n2 \quad \Leftrightarrow$

$y3 = x \otimes h3 + n3$

Frequency domain

$Y1 = X \cdot H1 + N1$

$Y2 = X \cdot H2 + N2$

$Y3 = X \cdot H3 + N3$

corr (y1,y1) $\Leftrightarrow$

$$Y1 \cdot Y1^* = X \cdot X^* \cdot H1 \cdot H1^* + N1 \cdot N1^* =$$
$$= P_{xx} \cdot H1 \cdot H1^* + P_{n1n1} =$$
$$= P_{y1y1}$$

(Auto Power Spectrum)

corr (y1,y2) $\Leftrightarrow$

$$Y1 \cdot Y2^* = X \cdot X^* \cdot H1 \cdot H2^* + N1 \cdot N2^* =$$
$$= P_{xx} \cdot H1 \cdot H2^* + P_{n1n2} =$$
$$= P_{y1y2}$$

(Cross Power Spectrum)

$$P_{y1y1} = Y1 \cdot Y1^* = X \cdot X^* \cdot H1 \cdot H1^* + N1 \cdot N1^*$$

$$P_{y1y2} = Y1 \cdot Y2^* = X \cdot X^* \cdot H1 \cdot H2^* + \cancel{N1 \cdot N2^*}$$

$$P_{y1y3} = Y1 \cdot Y3^* = X \cdot X^* \cdot H1 \cdot H3^* + \cancel{N1 \cdot N3^*}$$

$$P_{y2y2} = Y2 \cdot Y2^* = X \cdot X^* \cdot H2 \cdot H2^* + N2 \cdot N2^*$$

$$P_{y2y3} = Y2 \cdot Y3^* = X \cdot X^* \cdot H2 \cdot H3^* + \cancel{N2 \cdot N3^*}$$

$$P_{y3y3} = Y3 \cdot Y3^* = X \cdot X^* \cdot H3 \cdot H3^* + N3 \cdot N3^*$$

$$P_{y2y3} / P_{y1y3} = H2 / H1$$

$$P_{y2y2} / P_{y1y2} = H2 / H1 + N2 \cdot N2^* / P_{y1y2} = P_{y2y3} / P_{y1y3} + P_{n2n2} / P_{y1y2}$$

$$P_{n2n2} = P_{y2y2} - [P_{y1y2}/P_{y1y3}] \cdot P_{y2y3}$$

$$P_{n1n1} = P_{y1y1} - [P_{y1y3}/P_{y2y3}] \cdot P_{y2y1}$$

$$P_{n2n2} = P_{y2y2} - [P_{y1y2}/P_{y1y3}] \cdot P_{y2y3}$$

$$P_{n3n3} = P_{y3y3} - [P_{y3y2}/P_{y1y2}] \cdot P_{y3y1}$$

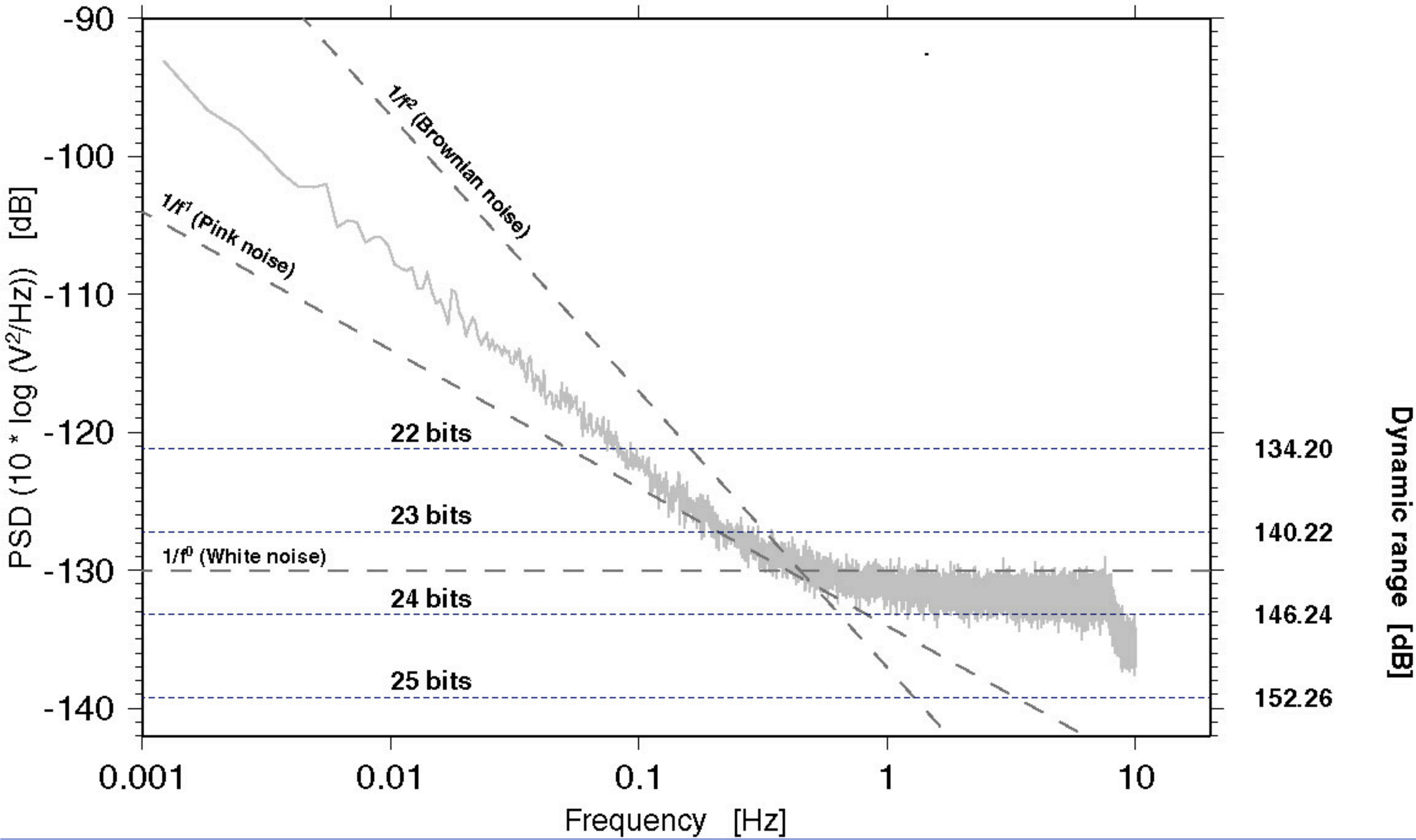
Cross power spectrum of outputs 1 and 3

Power spectrum of 3^{td} digitizer-noise

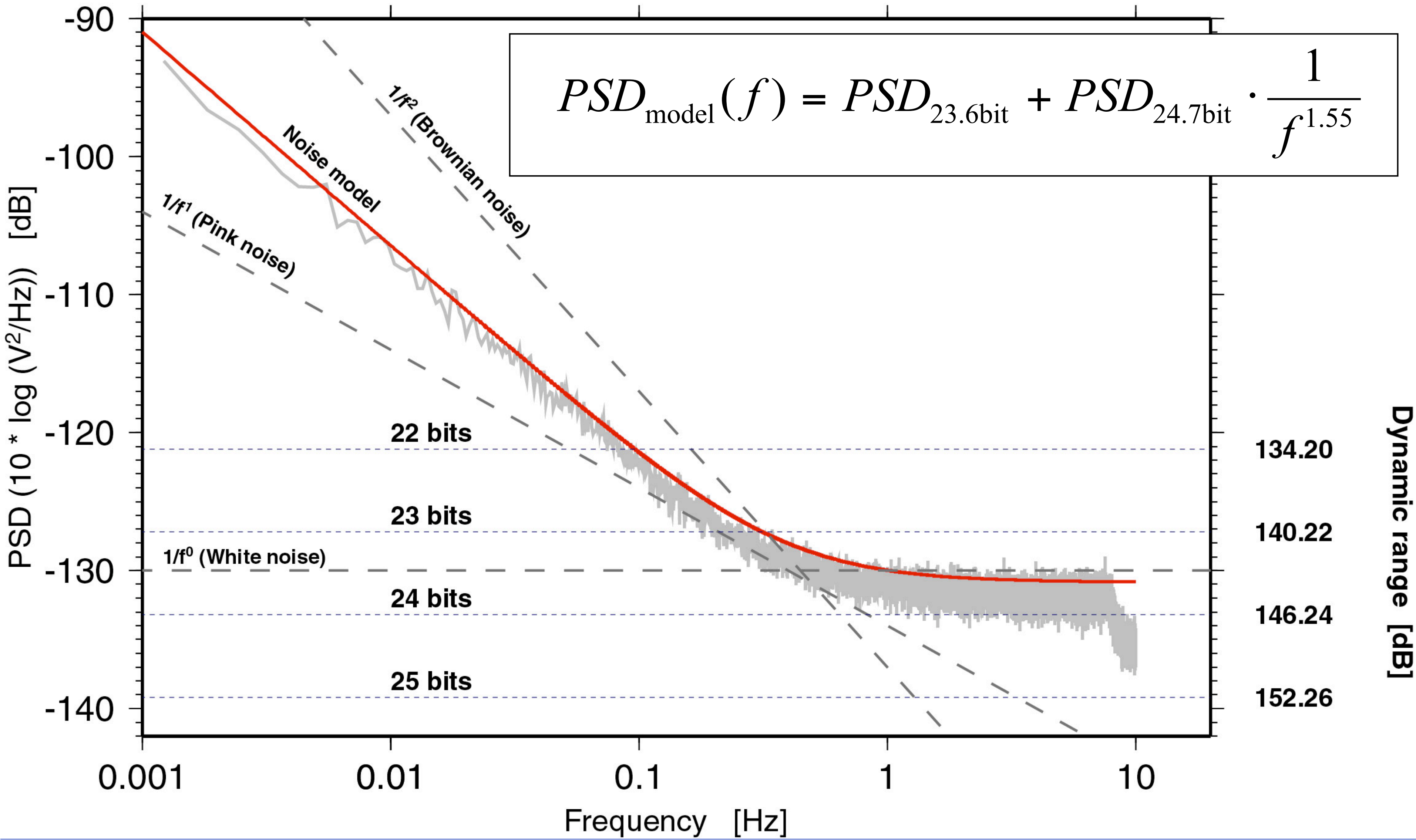
PSD estimation (Welsh, 1978):

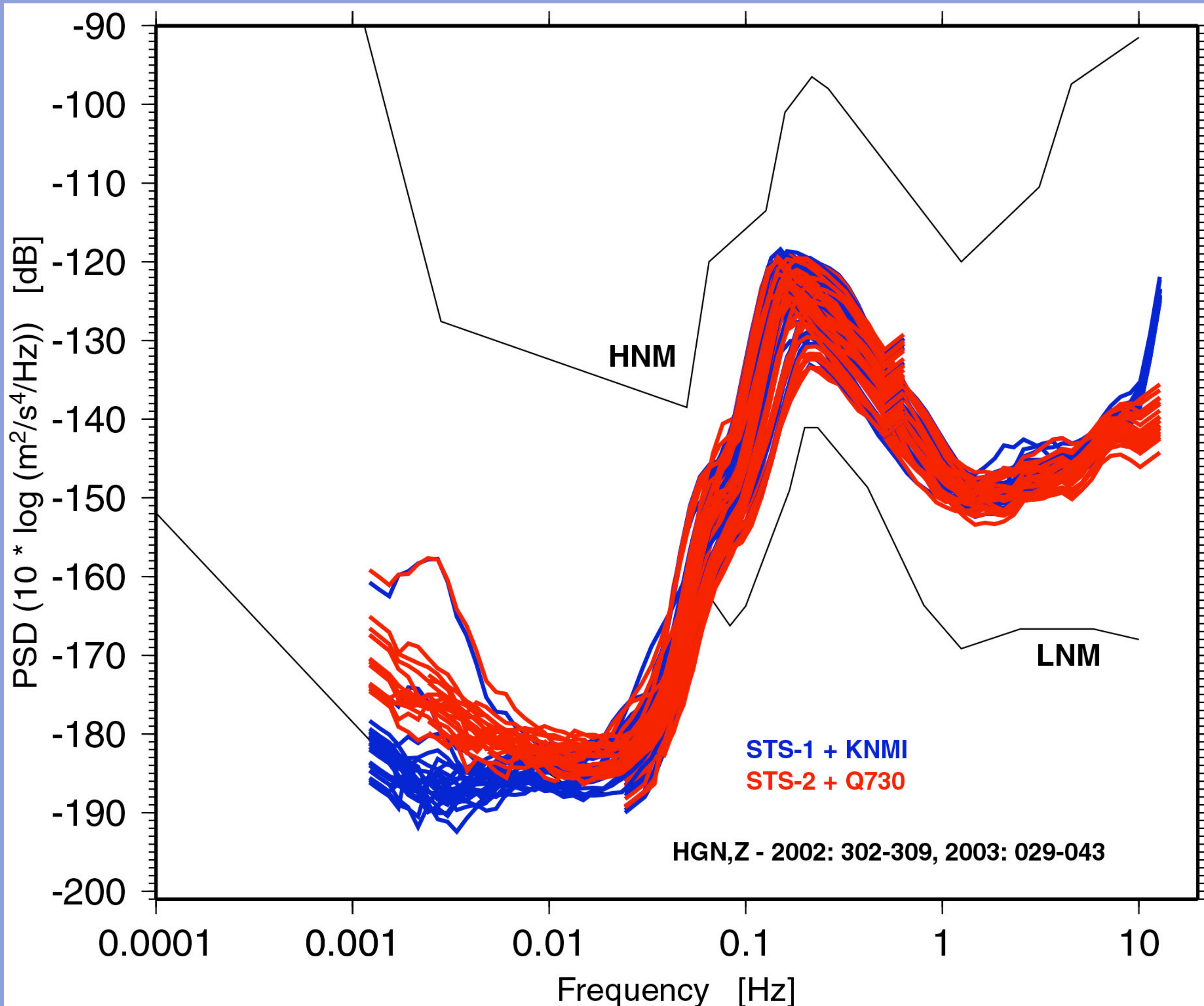
- 50 % overlapping time sections (of 2048 samples at 20 Hz)
- tapering (Hanning window)
- auto/cross correlation
- Fourier transform
- averaging over the number of time sections

PSD of self-noise Q4120
measured with common STS-2 vertical signal (@ 20 sps)

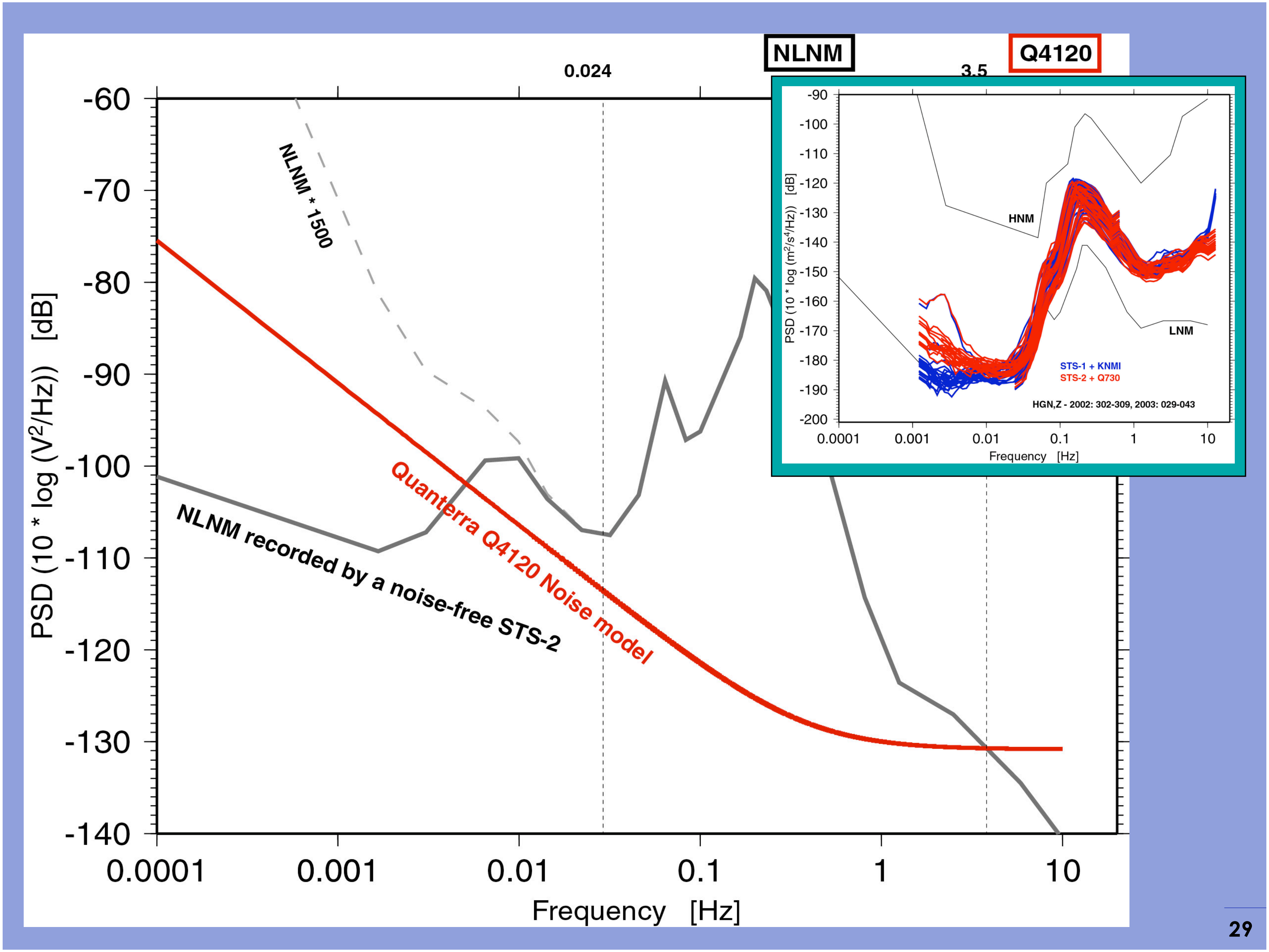


PSD of self-noise Q4120
measured with common STS-2 vertical signal (@ 20 sps)

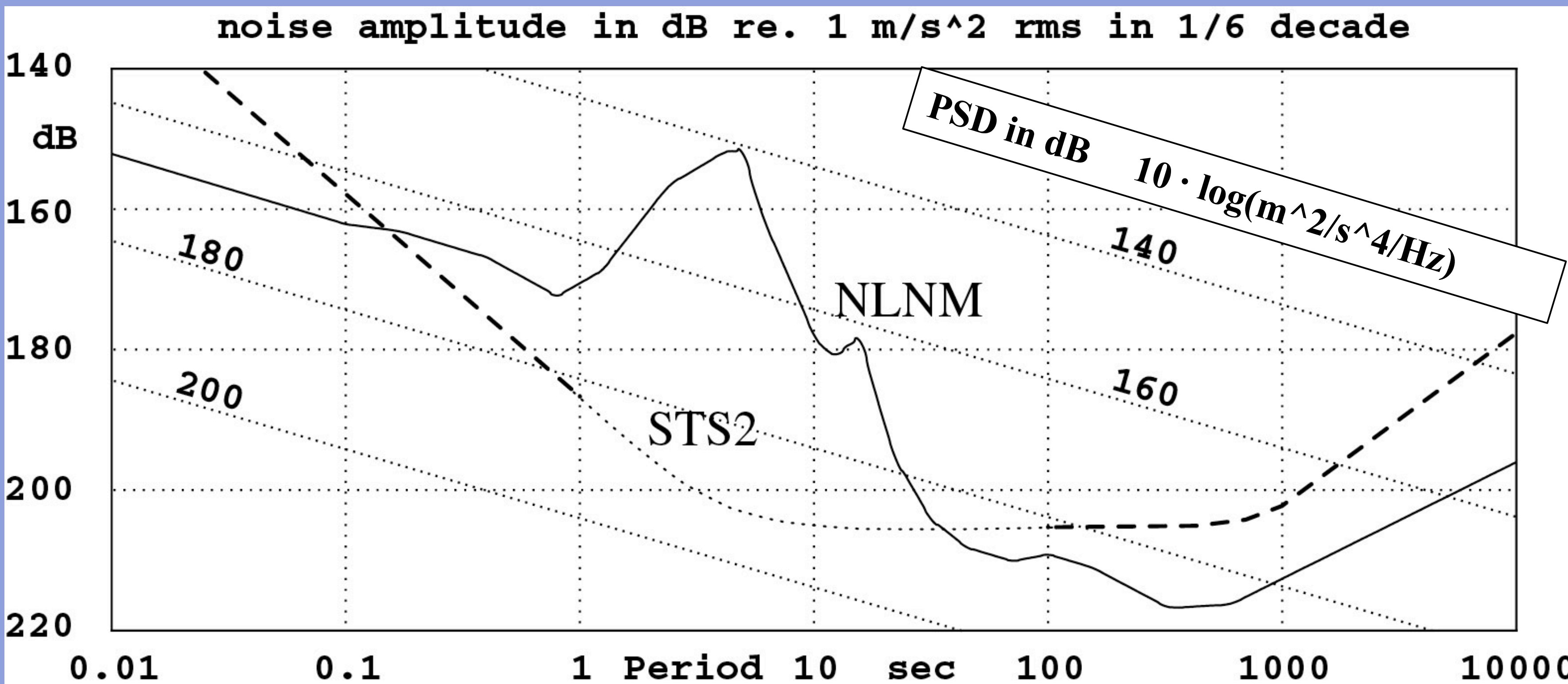




-90
-100
-110
-120
-130
-140
-150
-160
-170
-180
-190
-200
PSD (10 * log (m²/s⁴/Hz)) [dB]
0.0001
0.001
0.01
0.1
1
10
Frequency [Hz]
HNM
LNM
STS-1 + KNMI
STS-2 + Q730
HGN,Z - 2002: 302-309, 2003: 029-043

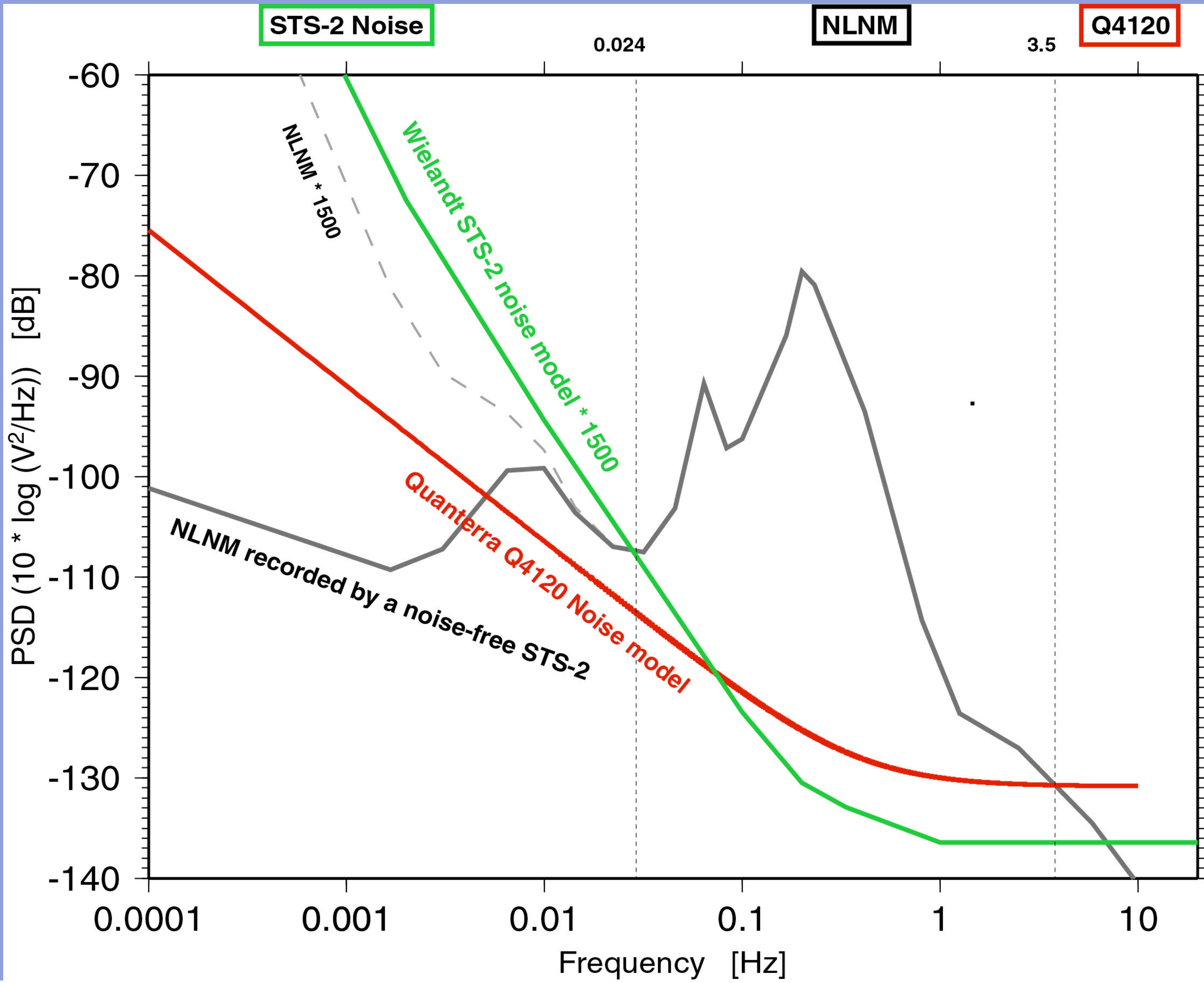

NLNM
Q4120
0.024
3.5
NLNM * 1500
Quanterra Q4120 Noise model
NLNM recorded by a noise-free STS-2
PSD (10 * log (V2/Hz)) [dB]
Frequency [Hz]
HNM
LNM
STS-1 + KNMI
STS-2 + Q730
HGN,Z - 2002: 302-309, 2003: 029-043
PSD (10 * log (m2/s4/Hz)) [dB]

From: Wielandt

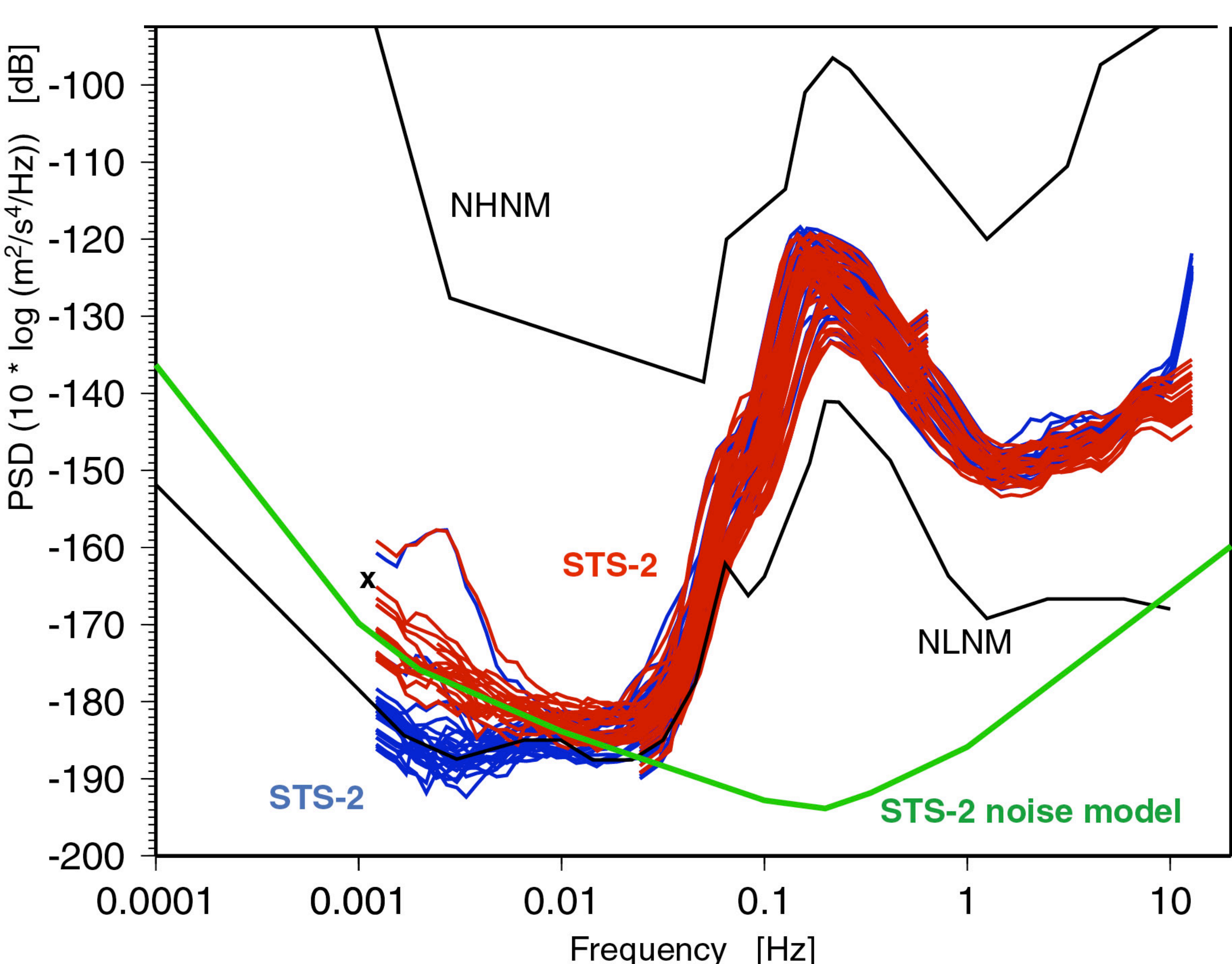


@1000 sec: RMS = 212 dB PSD = RMS^2/BW

PSD = 180 dB

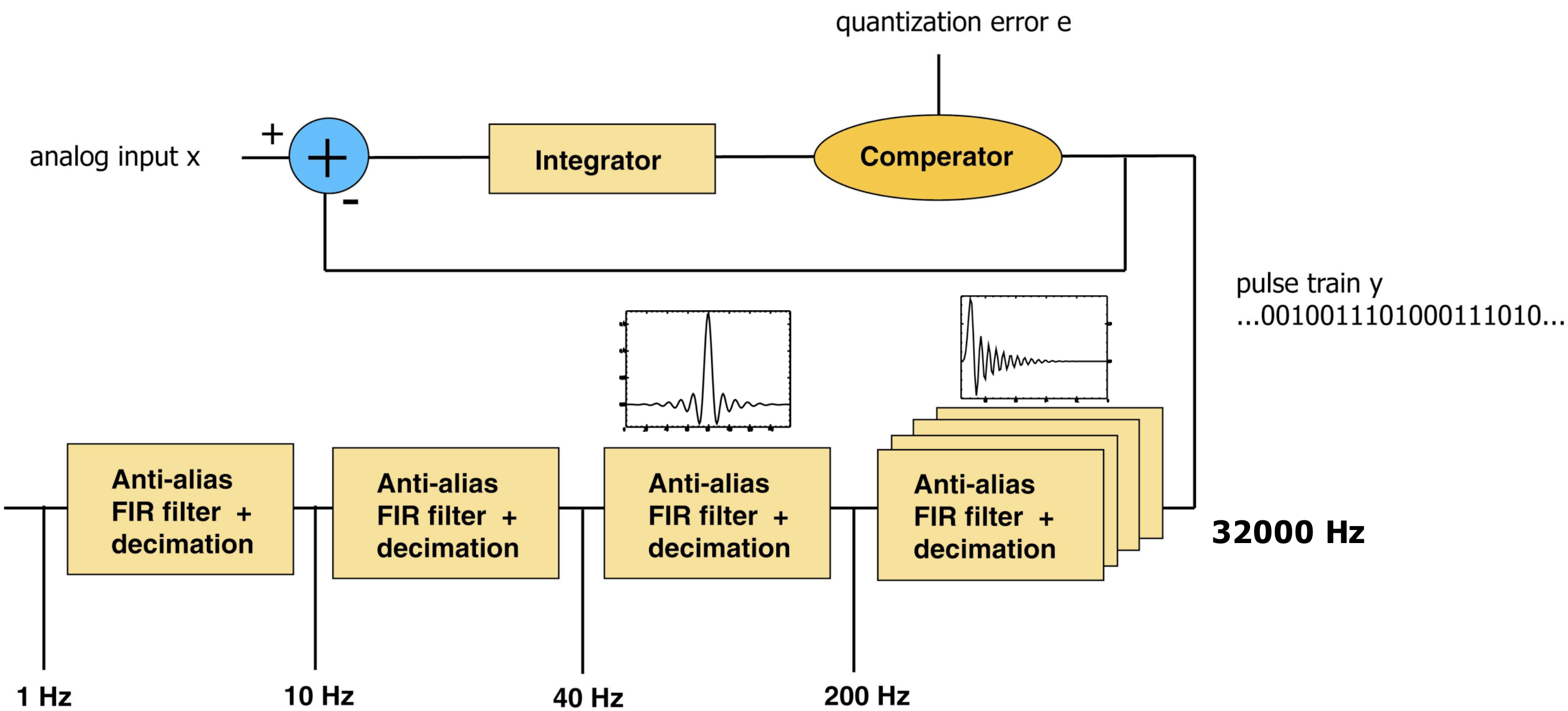


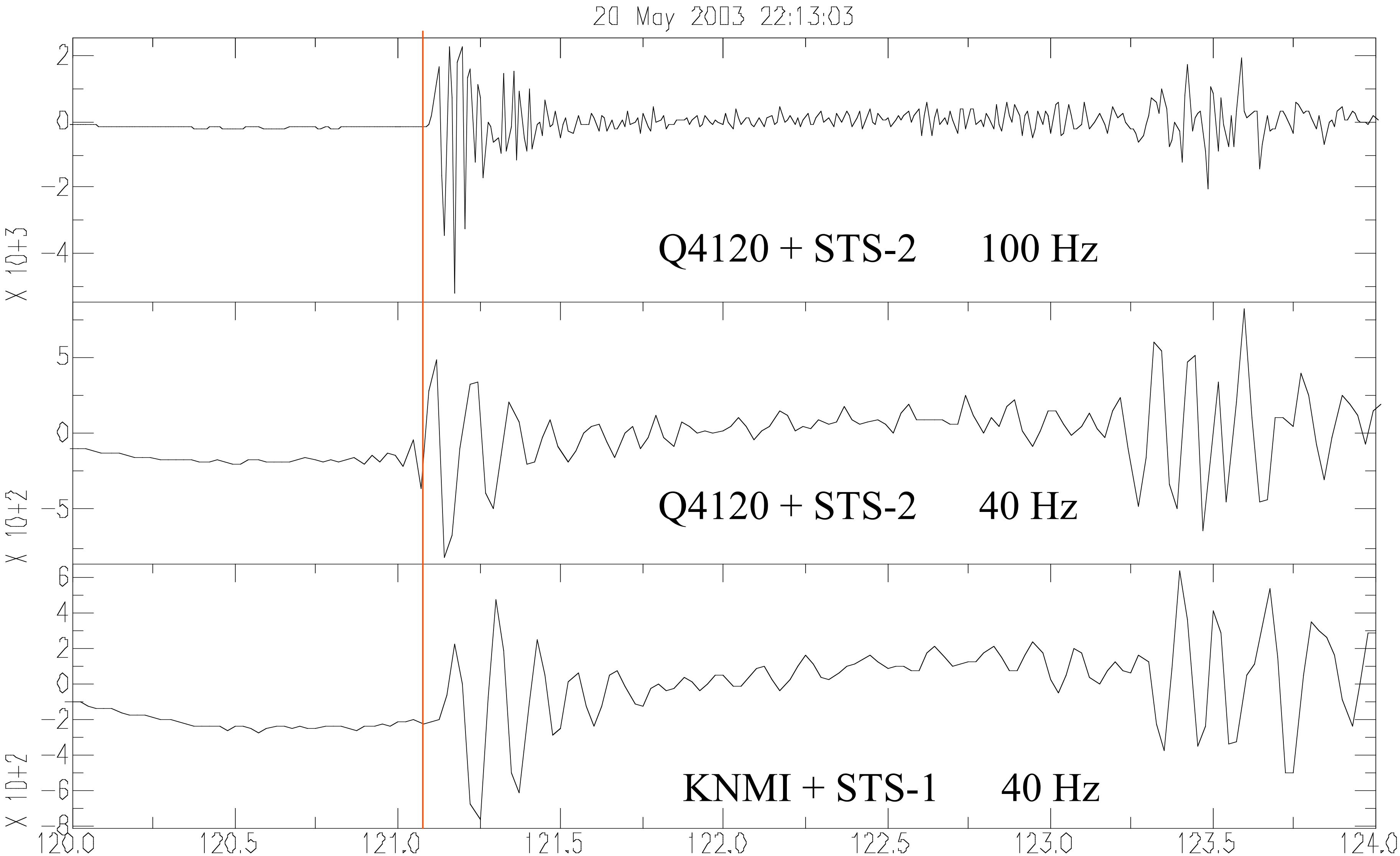
STS-2 Noise
NLNM
Q4120
0.024
3.5
-60
-70
-80
-90
-100
-110
-120
-130
-140
PSD (10 * log (V²/Hz)) [dB]
0.0001
0.001
0.01
0.1
1
10
Frequency [Hz]
NLNM * 1500
Wielandt STS-2 noise model * 1500
Quanterra Q4120 Noise model
NLNM recorded by a noise-free STS-2



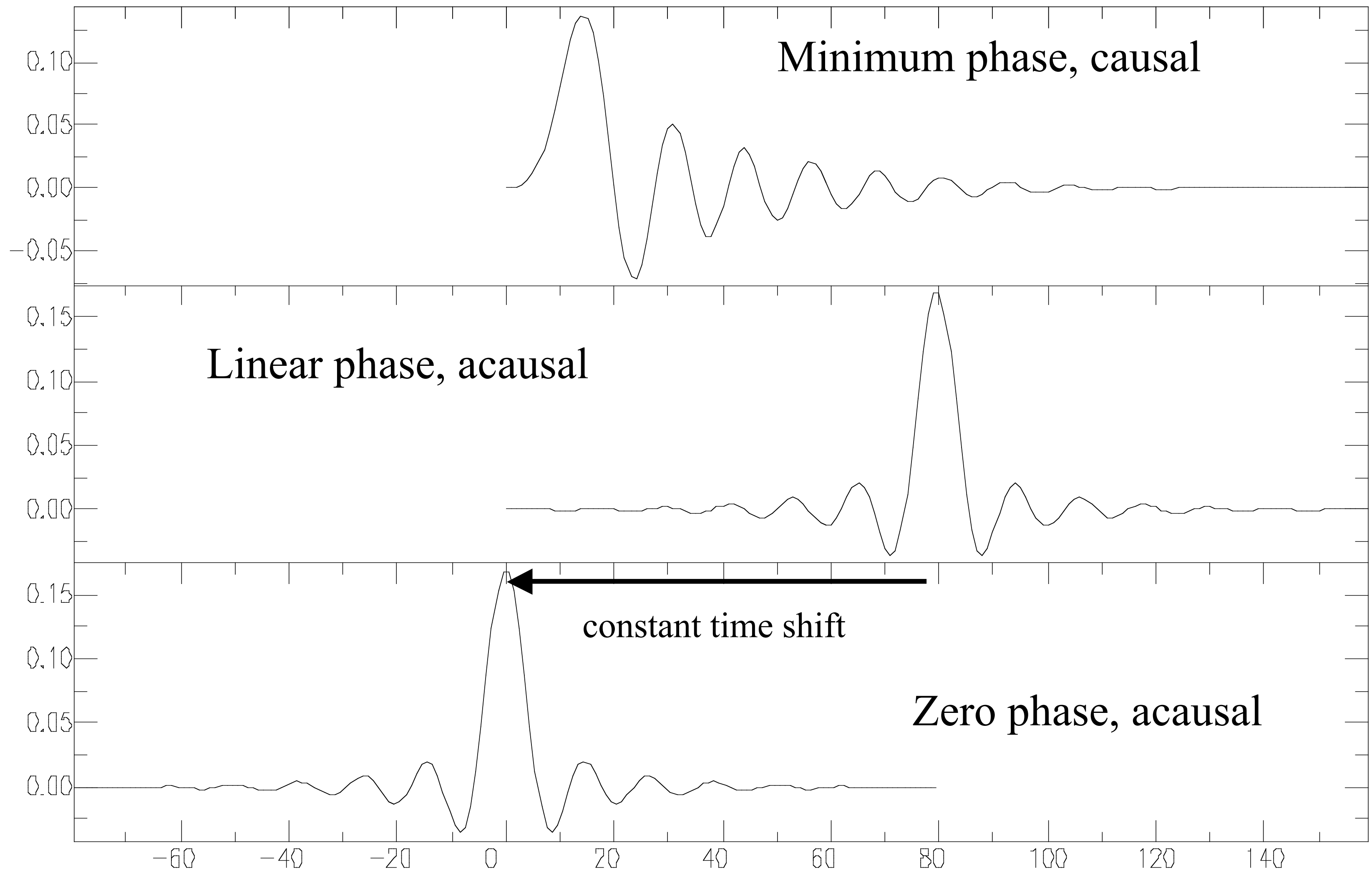
PSD (10 * log (m2/s4/Hz)) [dB]
-100
-110
-120
-130
-140
-150
-160
-170
-180
-190
-200
NHNM
NLNM
x
STS-2
STS-2
STS-2 noise model
0.0001
0.001
0.01
0.1
1
10
Frequency [Hz]

Datalogger data processing





20 May 2003 22:13:03
Q4120 + STS-2 100 Hz
Q4120 + STS-2 40 Hz
KNMI + STS-1 40 Hz
X 10+3
X 10+2
X 10+2
120.0
120.5
121.0
121.5
122.0
122.5
123.0
123.5
124.0


Minimum phase, causal
Linear phase, acausal
constant time shift
Zero phase, acausal
0.15
0.10
0.05
0.00
−0.05
−60
−40
−20
0
20
40
60
80
100
120
140

Removing acausal effects of the zero phase FIR filter

- FIR coefficients (polynomial coefficients) - Quanterra
- polynomial root finding (Jenkins/Traub, Muller, Newton) -

  Octave, Markus Lang, Mathematica
- replace maximum phase roots by minimum phase roots ($c \rightarrow 1/c^*$)
- get minimum phase equivalent FIR coefficients - Octave
- apply correction filter - Scherbaum

IIR filter: relation between coefficients and poles/zeros

$$y_k = \sum_{m=0}^{M} a_m \cdot x_{k-m}$$

time domain

a_m : filter coefficients

$$Y(z) = \sum_{m=0}^{M} a_m \cdot z^{-m} \cdot X(z)$$

z-domain

complex variable: $z = e^{s \cdot T}$

Numerator coefficients

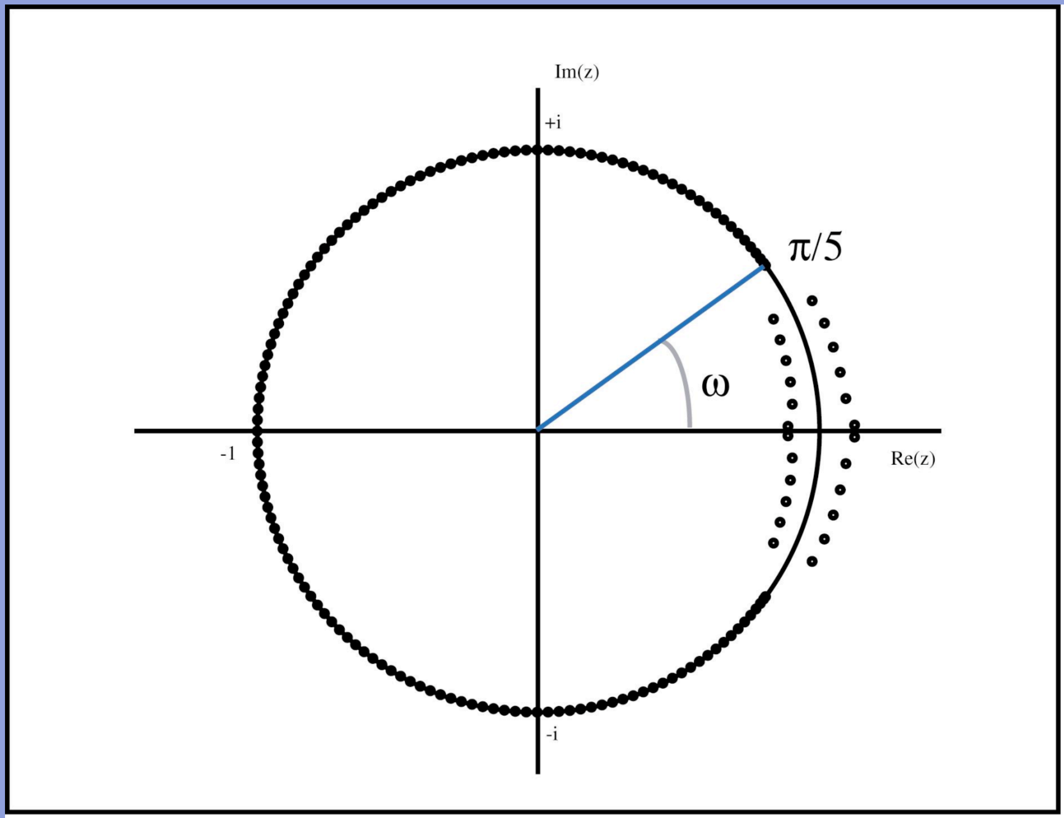
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{m=0}^{M} a_m \cdot z^{-m}}{\sum_{k=0}^{K} b_k \cdot z^{-k}} = \frac{a_0 \cdot \prod_{m=1}^{M} \left(1 - c_m \cdot z^{-1}\right)}{b_0 \cdot \prod_{k=1}^{K} \left(1 - d_k \cdot z^{-1}\right)}$$

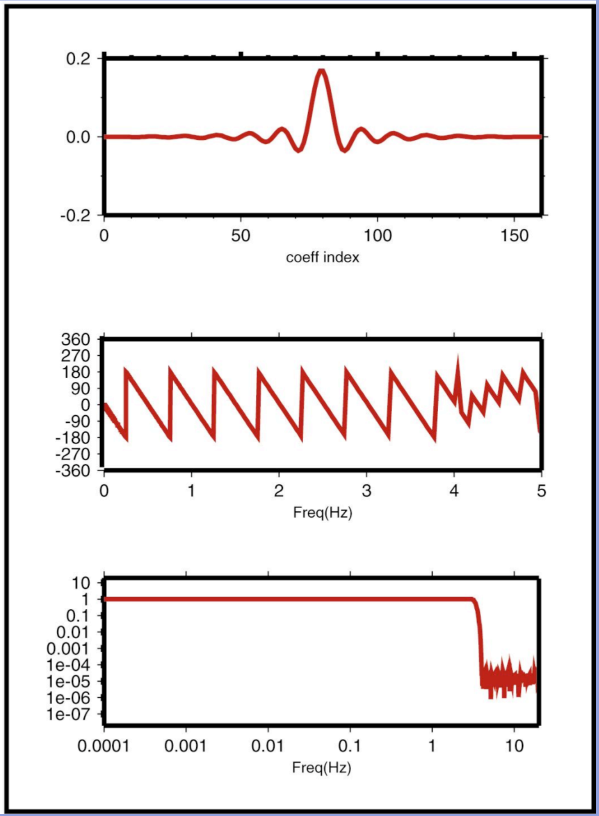
Denominator coefficients

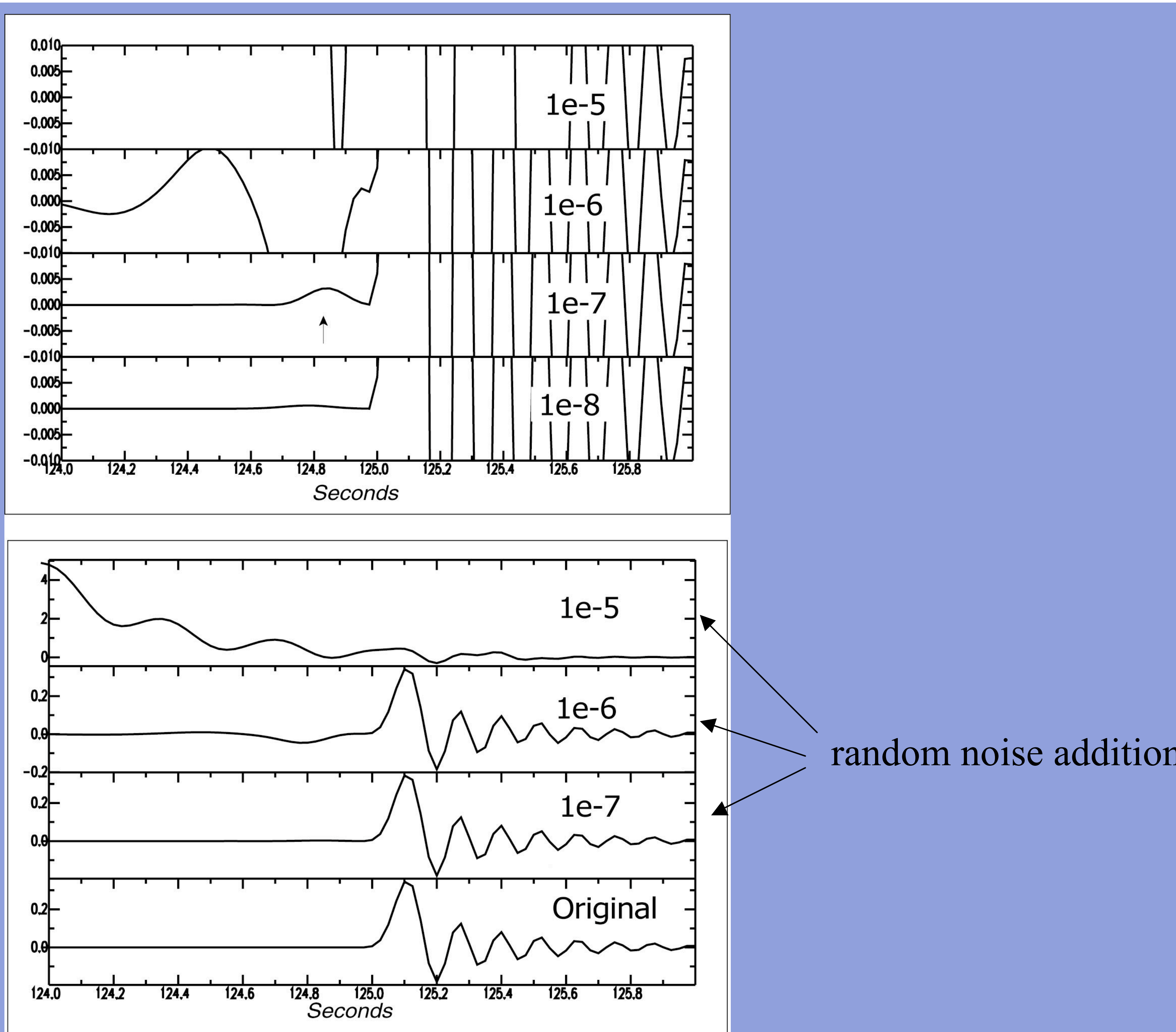
c_m : Roots of polynomial (zeros)

d_m : Roots of polynomial (poles)

Polynomial root finding







1e-5
1e-6
1e-7
1e-8
Seconds
1e-5
1e-6
1e-7
Original
Seconds
random noise addition