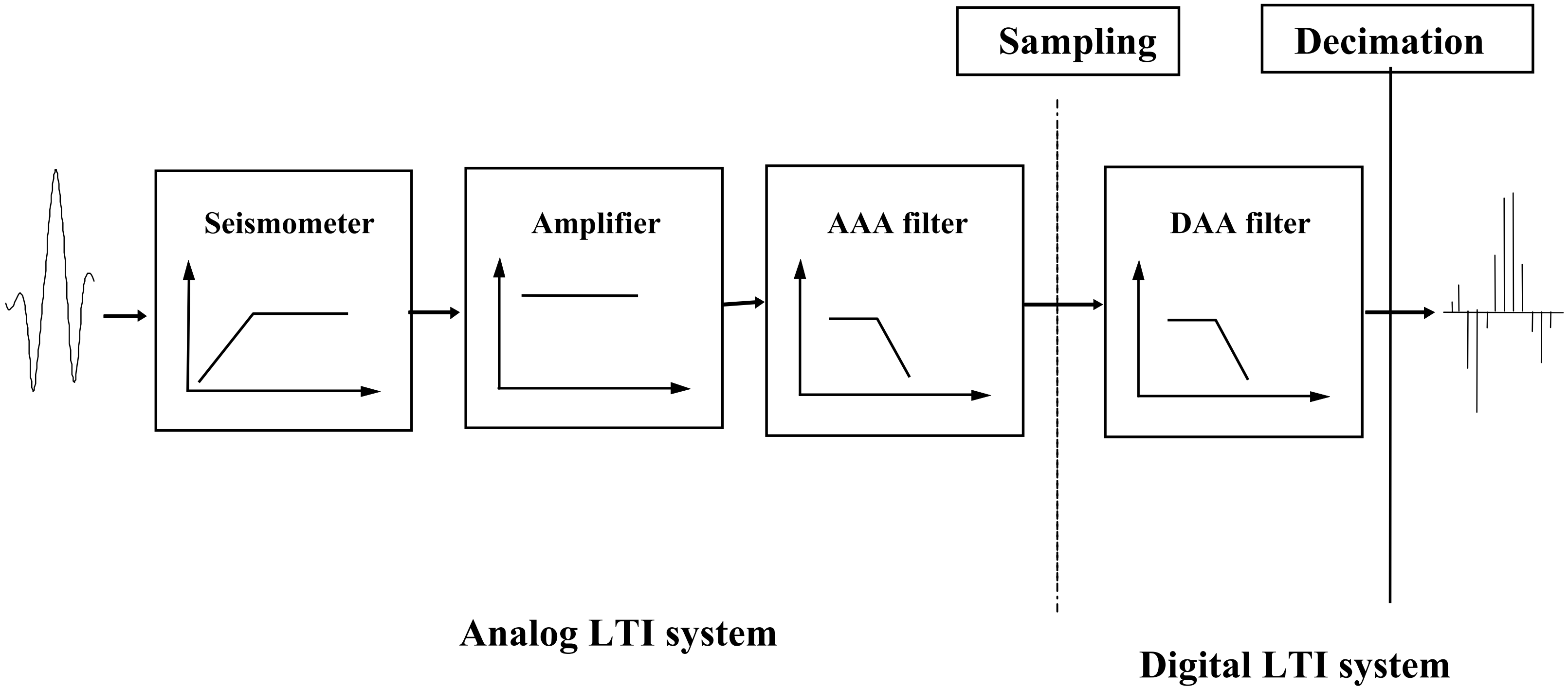

Sampling
Decimation
Seismometer
Amplifier
AAA filter
DAA filter
Analog LTI system
Digital LTI system

Filtering (Digital Systems)

Convolution of Sequences

$x[n]$ $\quad$ $h[n]$ $\quad$ $y[n] = \mathrm{h[n]} * \mathrm{x[n]}$

Discrete Fourier Transform (DFT)

$\widetilde{X}[k]$ $\quad$ $\widetilde{T}[k]$ $\quad$ $\widetilde{Y}[k] = \widetilde{T}[k]\widetilde{X}[k]$

z-Transform

$X[z]$ $\quad$ $T[z]$ $\quad$ $Y[z] = T[z]X[z]$

The z-Transform

$$Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = X(z)$$

Properties

- $x_1[n]*x_2[n] = \sum_{m=-\infty}^{\infty} x_1[m]x_2[n-m] \Leftrightarrow X_1(z) \cdot X_2(z)$ (convolution theorem)
- $x[n-n_0] \Leftrightarrow z^{-n_0}X(z)$ (shifting theorem)
- $x[-n] \Leftrightarrow X(1/z)$

Transfer Function

$$T(z) = \frac{Z\{y[n]\}}{Z\{x[n]\}} = \frac{Y(z)}{X(z)}$$

Rational Transfer Function $\Leftrightarrow$ **Linear Difference Equation**

$$T(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{l=0}^{M} b_l z^{-l}}{\sum_{k=0}^{N} a_k z^{-k}} \Leftrightarrow \sum_{k=0}^{N} a_k y[n-k] = \sum_{l=0}^{M} b_l x[n-l]$$

Recursive/Non-Recursive Filters

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{l=0}^{M} b_l x[n-l]$$

$$a_0 y[n] + \sum_{k=1}^{N} a_k y[n-k] = \sum_{l=0}^{M} b_l x[n-l]$$

$$y[n] = -\sum_{k=1}^{N} \frac{a_k}{a_0} y[n-k] + \sum_{l=0}^{M} \frac{b_l}{a_0} x[n-l]$$

recursive

(IIR Filters)

non-recursive

FIR Filters

($a_0 = 1$ and $a_k = 0$ for $k \geq 1$)

$$y[n] = \sum_{l=0}^{M} b_l x[n-l]$$

Rational Transfer Function

$$T(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{l=0}^{M} b_l z^{-l}}{\sum_{k=0}^{N} a_k z^{-k}}$$

Special Case: FIR filter $(a_0 = 1 \text{ and } a_k = 0 \text{ for } k \geq 1)$

$$T(z) = \sum_{l=0}^{M} b_l z^{-l} = b_0 \prod_{l=1}^{M} (1 - c_l z^{-1})$$

$$= z^{-M} \cdot b_0 \prod_{l=1}^{M} (z - c_l)$$

Linear Difference Equation

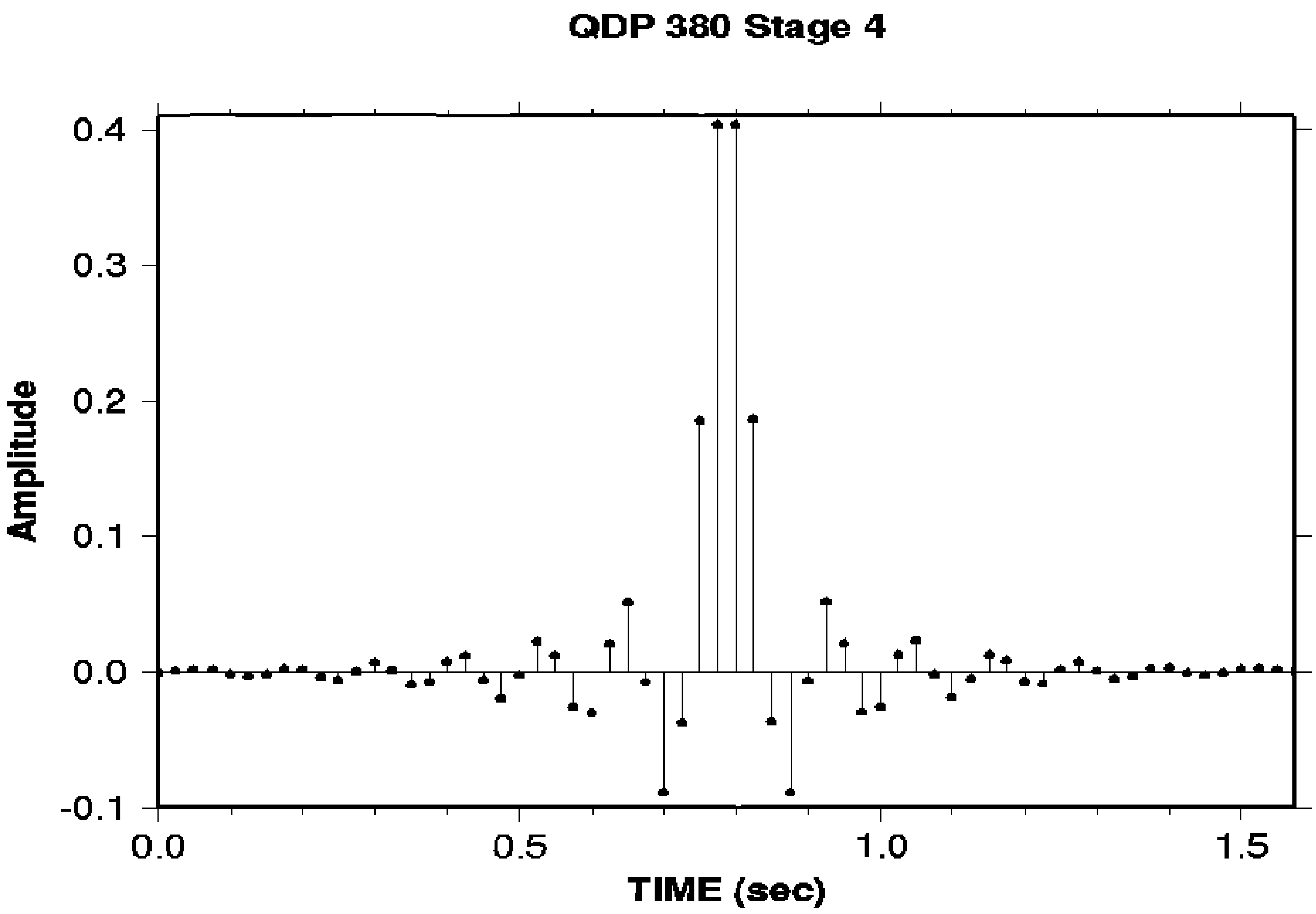
$$y[n] = \sum_{l=0}^{M} b_l x[n-l] = x[n]*\mathrm{b}[\mathrm{l}]$$

• FIR filters :

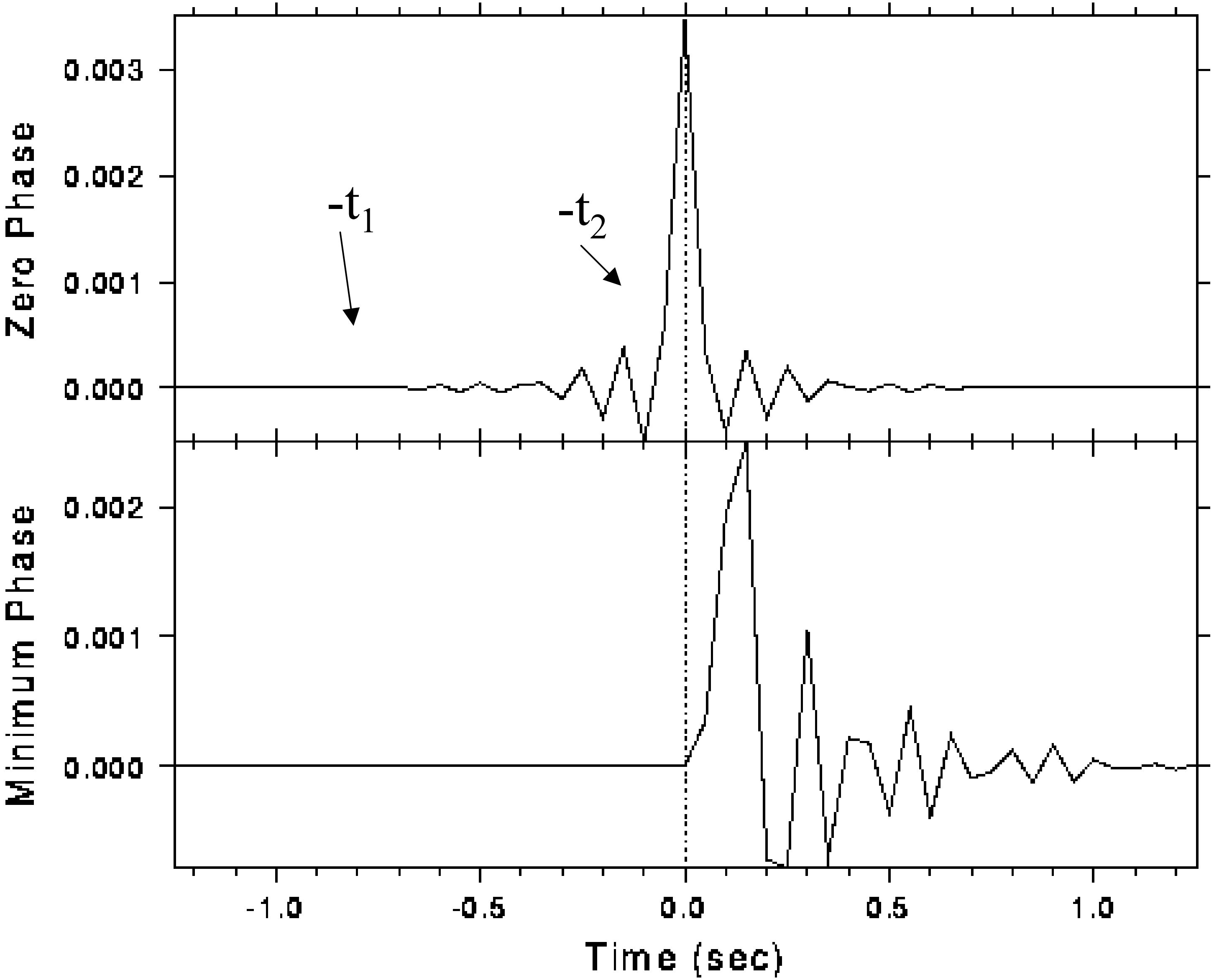
+ Always stable.
- Steep filters need many coefficients.
+ Both causal and noncausal filters can be implemented.
+ Filters with given specifications are easy to implement!

• IIR filters :

- Potentially unstable and subject to quantization errors.
+ Steep filters can easily be implemented with a few coefficients. Speed.
- Filters with given specifications are in general, difficult, if not impossible, to implement *exactly(!)*.



QDP 380 Stage 4
Amplitude
0.4
0.3
0.2
0.1
0.0
-0.1
0.0
0.5
1.0
1.5
TIME (sec)


0.003
0.002
0.001
0.000
Zero Phase
$-t_1$
$-t_2$
0.002
0.001
0.000
Minimum Phase
-1.0
-0.5
0.0
0.5
1.0
Time (sec)

How can we remove these effects?

Back to Transfer function:

Recursive/Non-Recursive Filters

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{l=0}^{M} b_l x[n-l]$$

$$a_0 y[n] + \sum_{k=1}^{N} a_k y[n-k] = \sum_{l=0}^{M} b_l x[n-l]$$

$$y[n] = -\sum_{k=1}^{N} \frac{a_k}{a_0} y[n-k] + \sum_{l=0}^{M} \frac{b_l}{a_0} x[n-l]$$

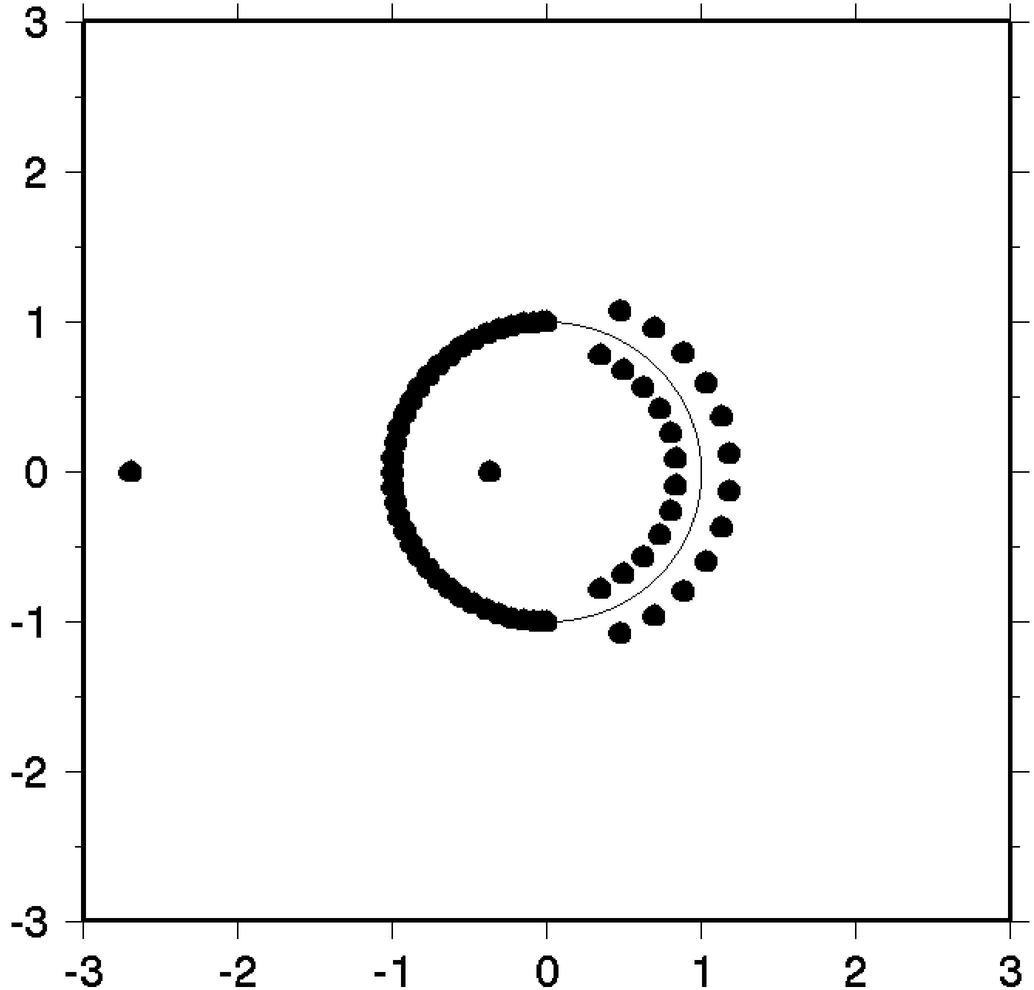
recursive → (IIR Filters)

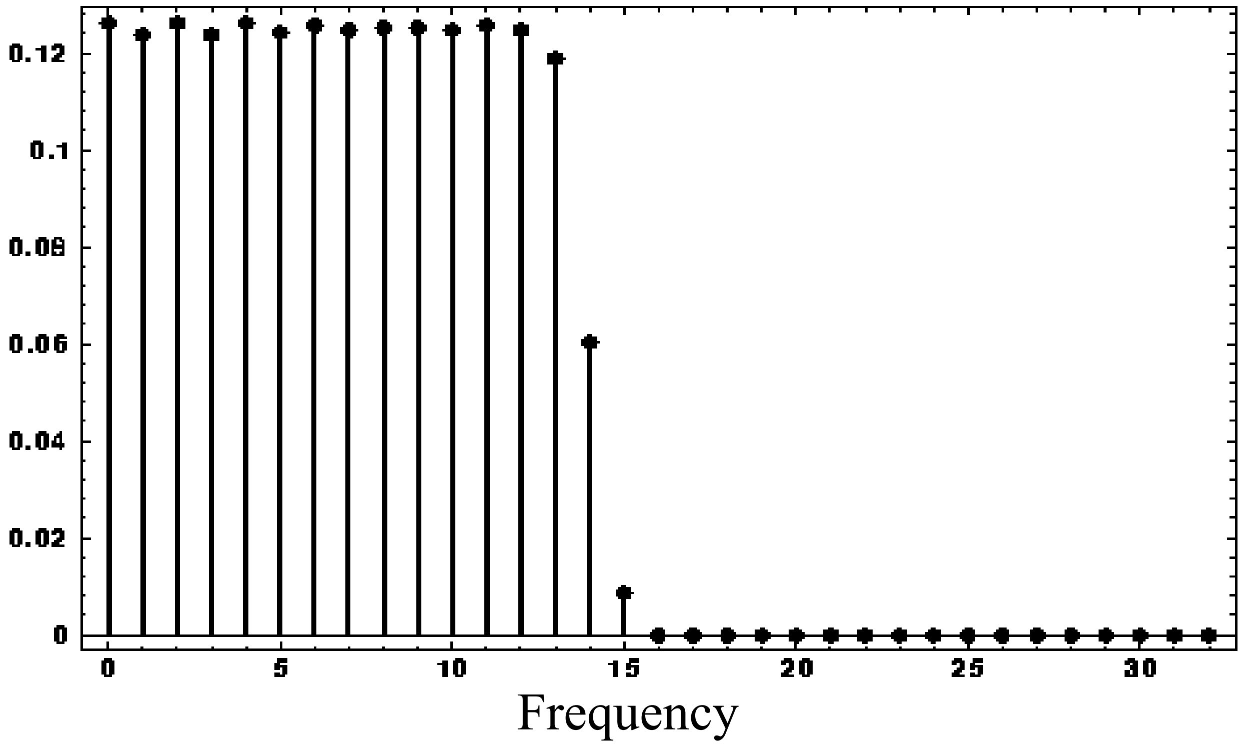
non-recursive → FIR Filters

($a_0 = 1$ and $a_k = 0$ for $k \ge 1$)

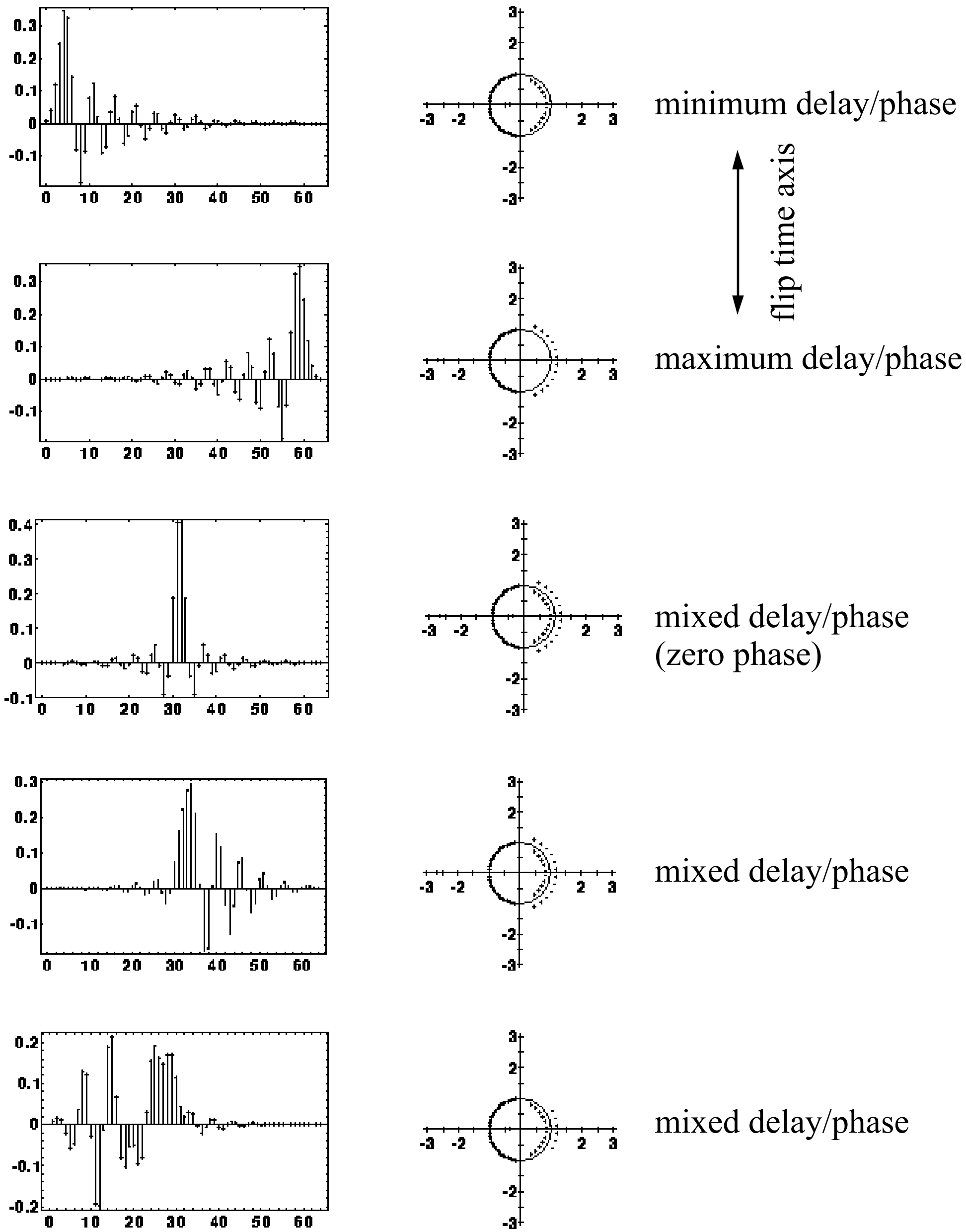
$$y[n] = \sum_{l=0}^{M} b_l x[n-l]$$

QDP 380 Stage 4




0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
Frequency

Waveform Properties and Root Positions



Zero Phase FIR Filter

Problem: Two-Sided IR

Cure: Change IR into Minimum Phase

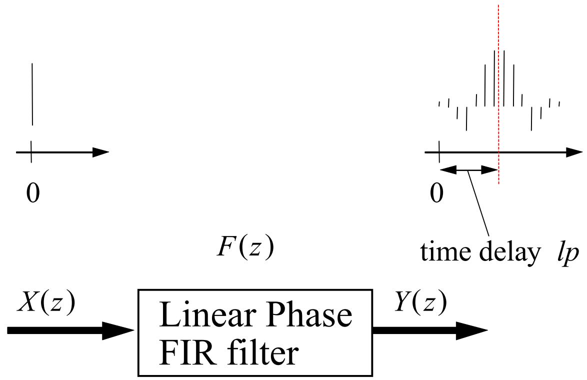
Methods:

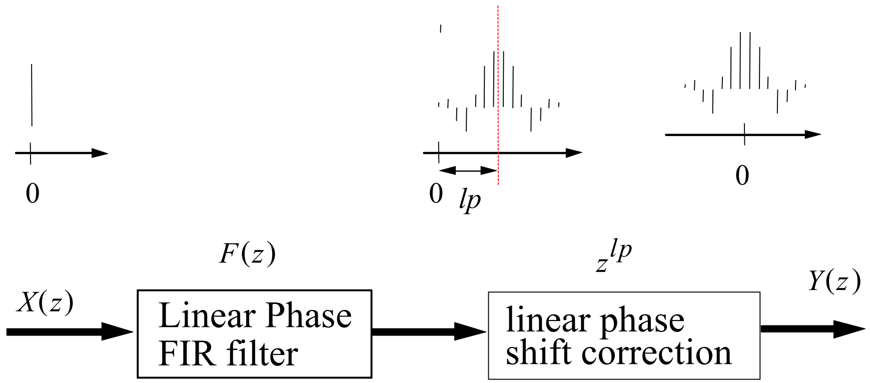
1) Add phase of Minimum Phase Filter to trace spectrum

2) Recursive Filtering of time inverted trace

Removing the acausal response of a Zero Phase FIR Filter

Linear Phase and Zero Phase Filter:





Linear-phase FIR Filter: $F(z)$

Zero-phase FIR Filter: $F(z) = F(z) \cdot z^{lp}$

z^{lp} = Time delay correction by lp samples

General rule:
Any filter can be expressed by convolution of its minimum phase and maximum phase component.

roots within UC (minimum phase)   roots outside UC (maximum phase)

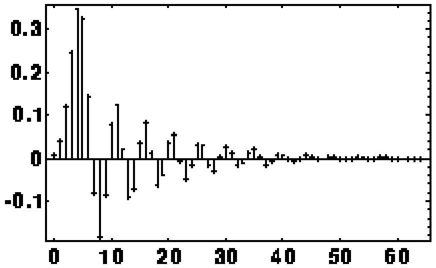
=> Linear phase FIR Filter: $F(z) = F_{max}(z) \cdot F_{min}(z)$

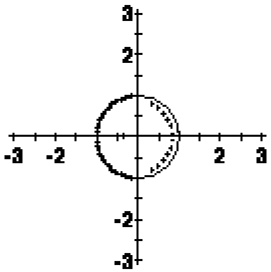
maximum phase component -> left-sided (acausal) component
minimum phase component -> right-sided (causal) component

Removal of acausal response:

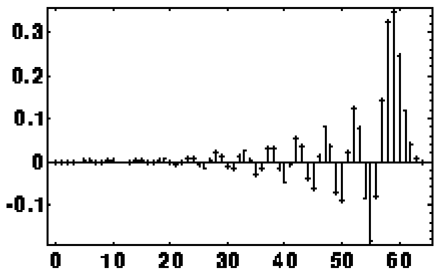
Replace maximum phase component $F_{max}(z)$ by its minimum phase equivalent $MinPhase\{F_{max}(z)\}$.

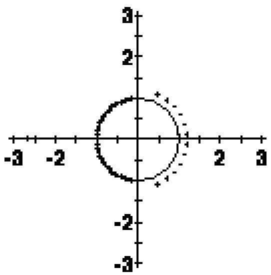
$$MinPhase\{F_{max}(z)\} =$$





minimum phase





maximum phase

Answer: flip maximum phase component in time.
In terms of z-transform:

=> $MinPhase\{F_{max}(z)\} = F_{max}(1/z)$

z-transform representation of digital seismogram

LP FIR

LP correction

$$\widetilde{Y}(z) = F(z) \cdot z^{lp} \cdot \widetilde{X}(z)$$

digital seismogram after FIR filtering but before decimation

0P FIR

digital seismogram before FIR filtering

$\widetilde{x}[n]$ = unfiltered digital seismogram

$\widetilde{X}(z)$ = z-transform of the input signal $\widetilde{x}[n]$

$F(z)$ = z-transform of the linear phase FIR filter

$\widetilde{y}[n]$ = filtered digital seismic trace before decimation[1]

$\widetilde{Y}(z)$ = z-transform of $\widetilde{y}[n]$

---

[1] This is a fictitious signal since decimation is commonly done while filtering.

$$F(z) \cdot z^{lp}$$

corresponds to a zero phase filter in which the linear phase component of $F(z)$ is corrected.

In practice: Treat time shift z^{lp} separately from F(z)

Removing the maximum phase component of a FIR filter

Principle:

$F_{max}(z)$ -> $MinPhase\{F_{max}(z)\}$

'Corrected' seismogram $Y(z)$: $$Y(z) = \frac{1}{F_{max}(z)} \cdot F_{max}(1/z) \cdot \tilde{Y}(z)$$

Problem: Since $F_{max}(z)$ has only zeros outside the unit circle, $1/F_{max}(z)$ will have poles outside the unit circle.

<u>Solution:</u> Flip time axis.

$$Y(1/z) = \frac{1}{F_{max}(1/z)} \cdot F_{max}(z) \cdot \tilde{Y}(1/z)$$

Impulse response corresponding to $1/\ F_{max}(z)$ becomes a stable causal sequence in nominal time and the deconvolution of the maximum phase component $F_{max}(z)$ poses no stability problems.

The difference equation

$$F_{max}(1/z) \cdot Y(1/z) = F_{max}(z) \cdot \tilde{Y}(1/z)$$

Rewrite to

$$A'(z) \cdot Y'(z) = B'(z) \cdot X'(z)$$

$$A'(z) \Leftrightarrow F_{max}(1/z) \qquad Y'(z) \Leftrightarrow Y(1/z)$$

$$B'(z) \Leftrightarrow F_{max}(z) \qquad X'(z) \Leftrightarrow \tilde{Y}(1/z)$$

Written as convolution sum:

$$\sum_{k=-\infty}^{\infty} a'[k] \cdot y'[i-k] = \sum_{l=-\infty}^{\infty} b'[l] \cdot x'[i-l]$$

Assumption: $F(z)$ contains mx zeros outside the unit circle
=> wavelets $a'[k]$ and $b'[k]$ will be of length $mx + 1$.

$$\sum_{k=0}^{mx} a'[k] \cdot y'[i-k] = \sum_{l=0}^{mx} b'[l] \cdot x'[i-l]$$

Rearrange to

$$y'[i] \cdot a'[0] + \sum_{k=1}^{mx} a'[k] \cdot y'[i-k] = \sum_{l=0}^{mx} b'[l] \cdot x'[i-l]$$

which is equivalent to

$$y'[i] = -\sum_{k=1}^{mx} \frac{a'[k]}{a'[0]} \cdot y'[i-k] + \sum_{l=0}^{mx} \frac{b'[l]}{a'[0]} \cdot x'[i-l]$$

This is

$$a[k] = -\frac{a'[k]}{a'[0]} = \frac{f_{max}[mx-k]}{f_{max}[mx]} \qquad \text{for } k = 1 \text{ to } mx$$

and

$$b[l] = \frac{b'[l]}{a'[0]} = \frac{f_{max}[l]}{f_{max}[mx]} \qquad \text{for } l = 0 \text{ to } mx$$

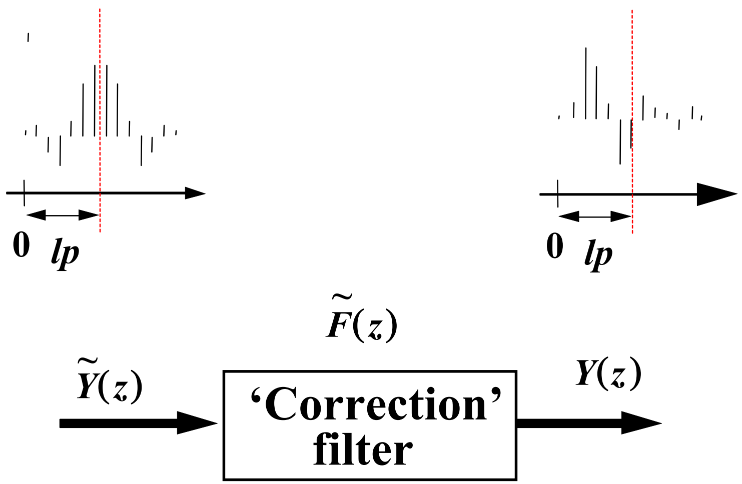
The convolution sum

$$y'[i] = \sum_{k=1}^{mx} a[k] \cdot y'[i-k] + \sum_{l=0}^{mx} b[l] \cdot x'[i-l]$$

$y'[i]$ = the time reversed ‘corrected’ sequence.

To obtain $y[i]$ flip $y'[i]$ back in time.

<u>'Corrected' seismogram:</u> $Y(z) = \tilde{F}(z) \cdot \tilde{Y}(z)$



$\tilde{F}(z)$ = changes the linear phase filter $F(z)$ into a minimum phase filter

=> onset of output signal is advanced by *lp* samples.

To account for this time shift in the corrected seismogram:

a) change time tag of ‘corrected’ trace

or

b) delay ‘corrected’ trace by *lp* samples.

What is needed for 'correction'?

$mx + 1$ coefficients of the maximum phase portion of the a linear phase FIR filter

How to get maximum phase component?

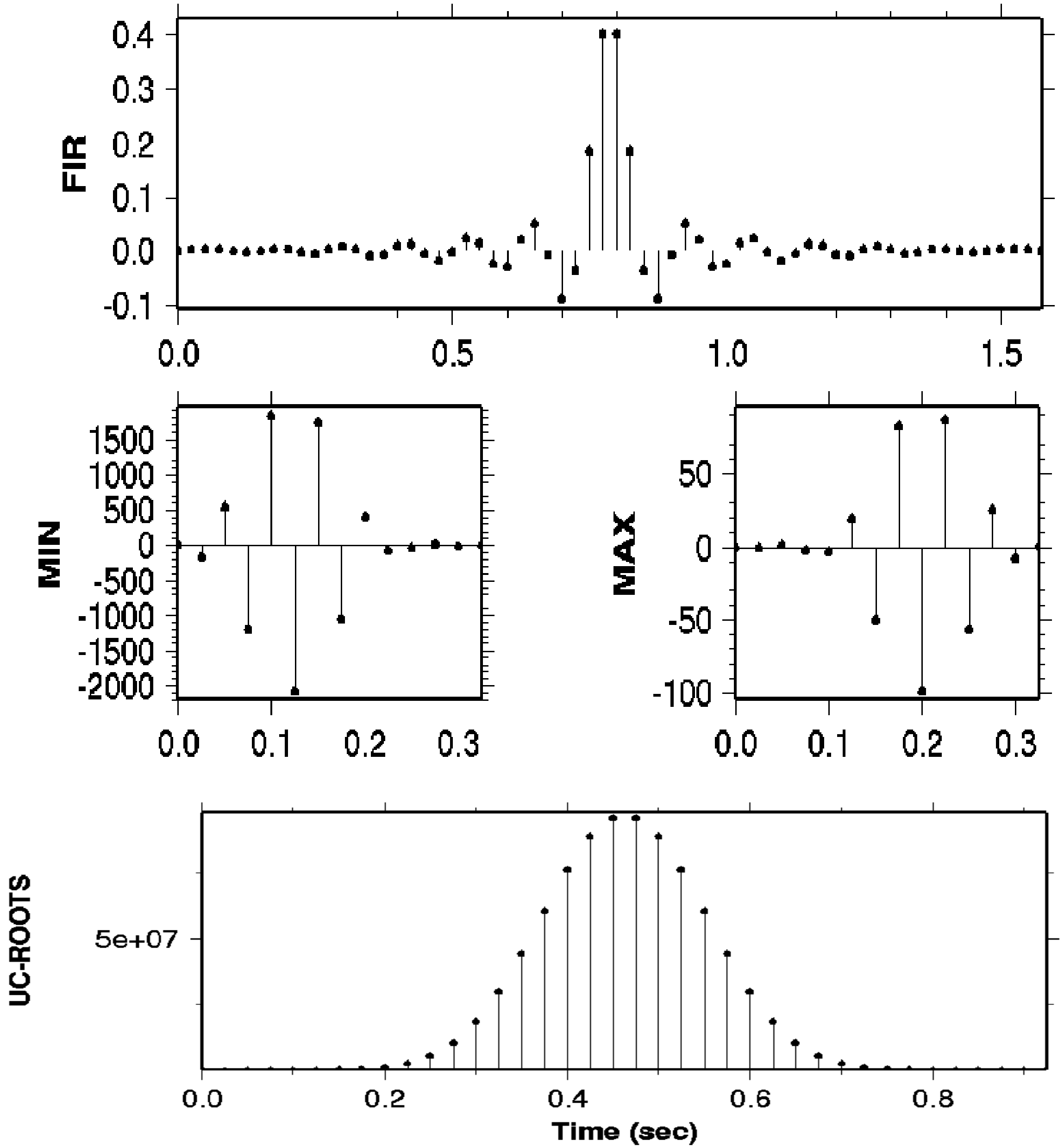
$$F(z) = \sum_{l=0}^{m} b_l z^{-l} = b_0 \prod_{l=1}^{m} (1 - c_l z^{-l})$$

mn zeros inside UC (c_i^{min}) <=> $F_{min}(z)$

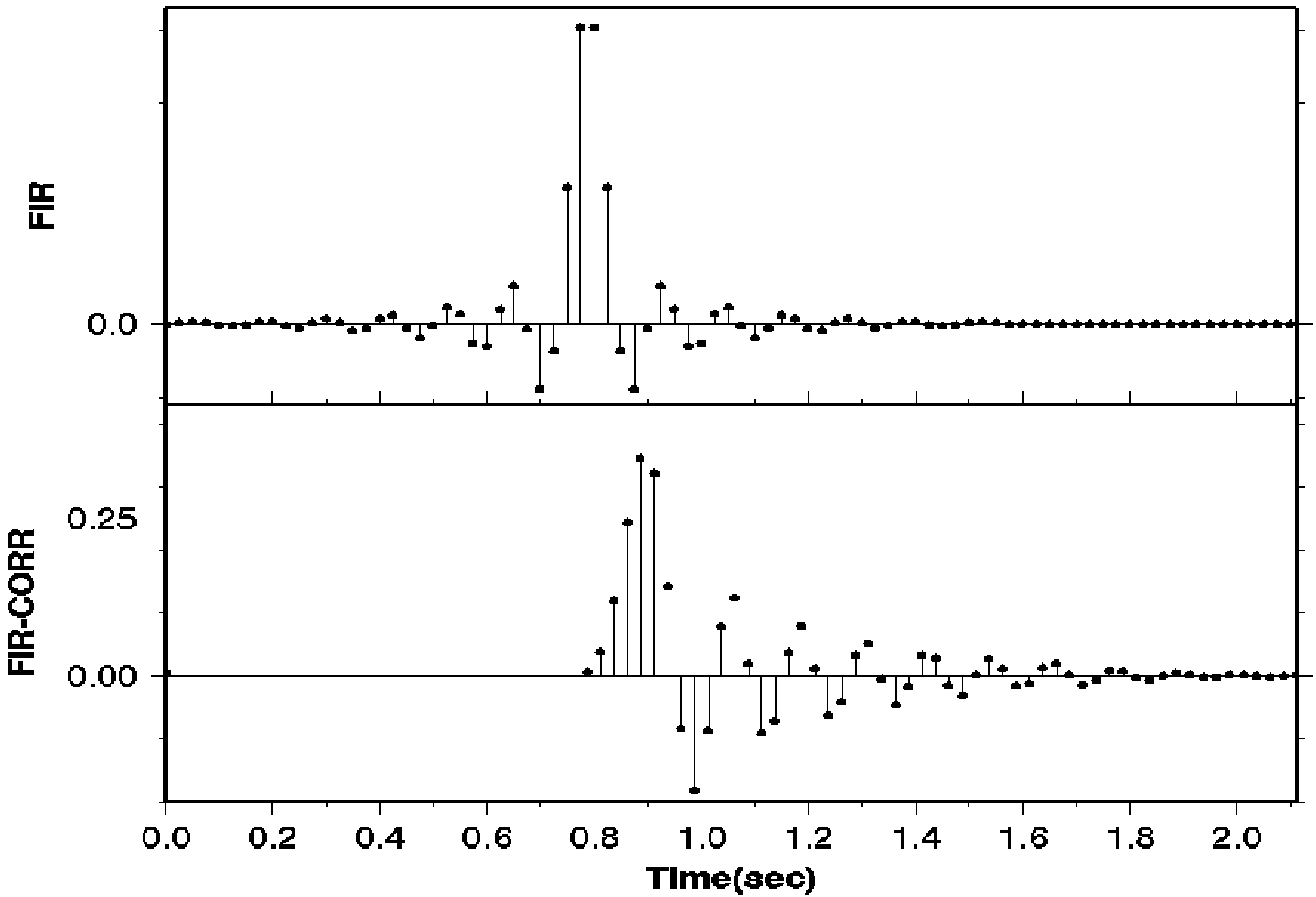
mx zeros outside UC (c_i^{max}) <=> $F_{max}(z)$

$$F_{max}(z) = \sum_{i=0}^{mx} f_i^{max} z^{-i} = b_0 \prod_{i=1}^{mx} (1 - c_i^{max} z^{-i})$$

From zeros ouside UC -> polynomial $F_{max}(z)$.
$f_{max}[l]$ for $l = 0$ to mx: coefficients of polynomial $F_{max}(z)$.

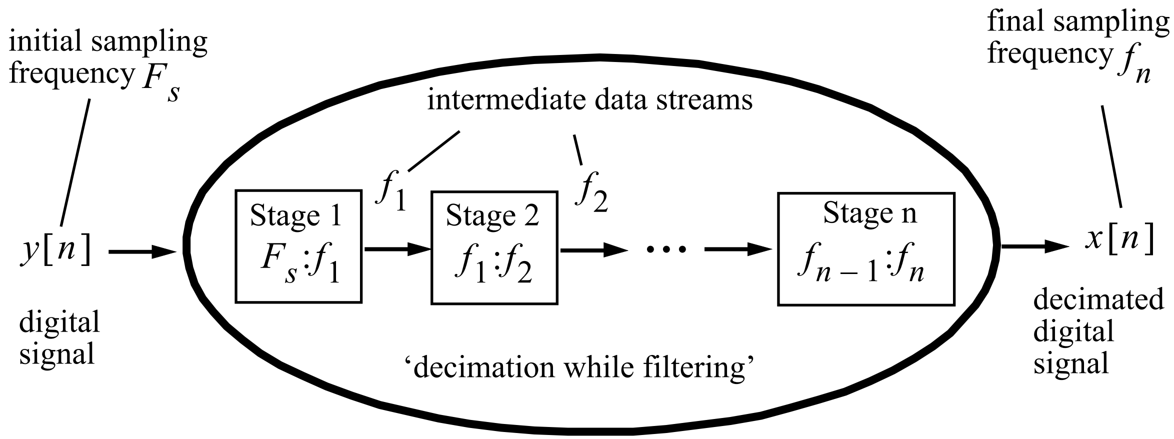


FIR
0.4
0.3
0.2
0.1
0.0
-0.1
0.0
0.5
1.0
1.5
MIN
1500
1000
500
0
-500
-1000
-1500
-2000
0.0
0.1
0.2
0.3
MAX
50
0
-50
-100
0.0
0.1
0.2
0.3
UC-ROOTS
5e+07
0.0
0.2
0.4
0.6
0.8
Time (sec)


FIR
0.0
FIR-CORR
0.25
0.00
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Time(sec)

Correction Procedure

Decimation stages



Full Correction (not always necessary):

interpolation -> f_{n-1} -> correction for FIR filter stage n

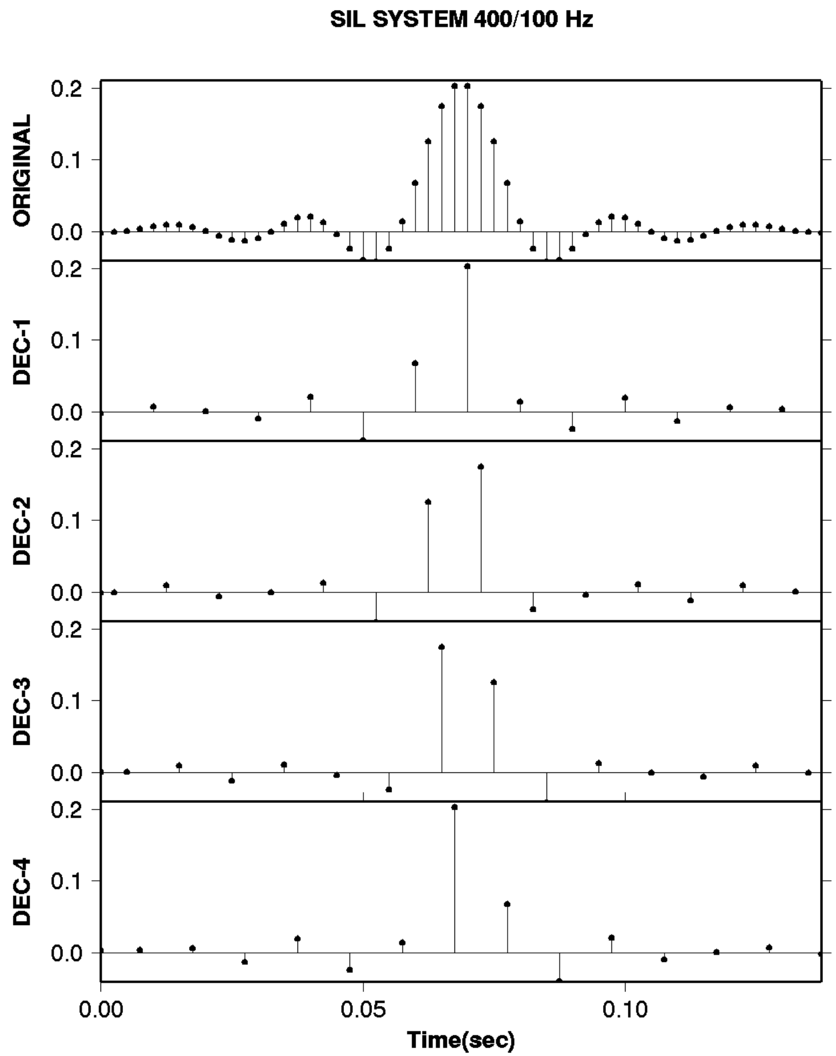
interpolation -> f_{n-2} -> correction for FIR filter stage n-1

⋮

interpolation -> f_1 -> correction for FIR filter stage 2

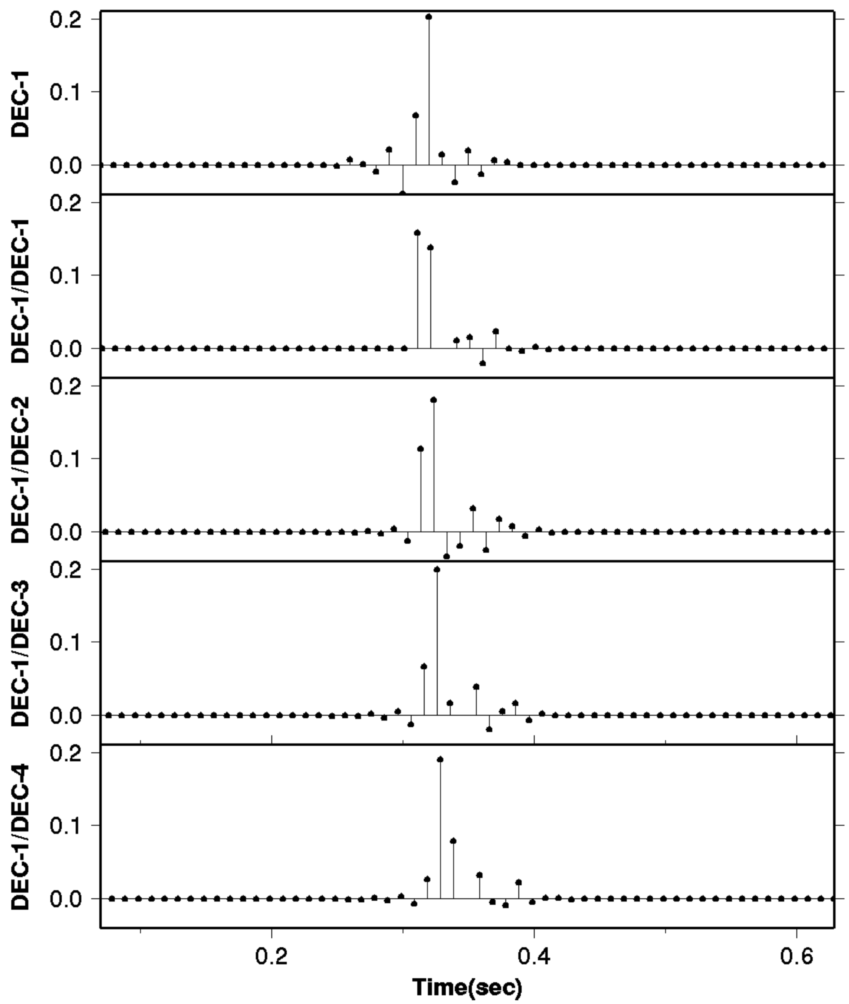
interpolation -> F_s -> correction for FIR filter stage 1

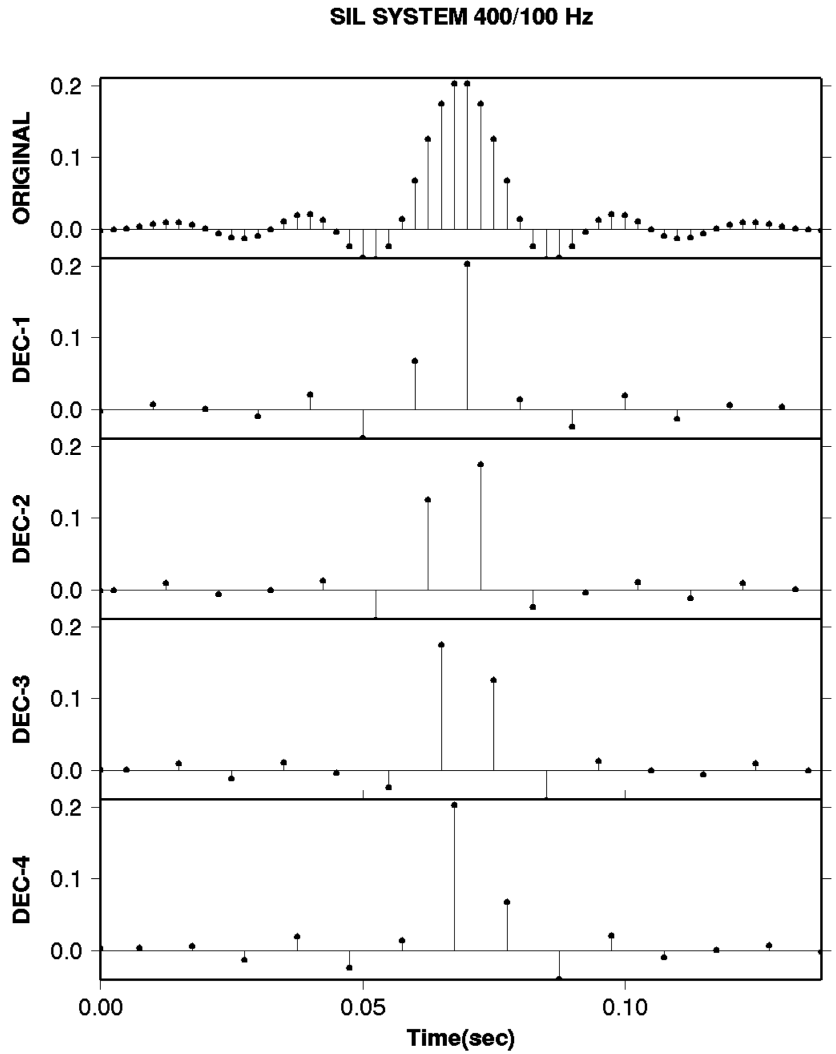
finally: decimation back to f_n



SIL SYSTEM 400/100 Hz
ORIGINAL
DEC-1
DEC-2
DEC-3
DEC-4
0.2
0.1
0.0
0.00
0.05
0.10
Time(sec)

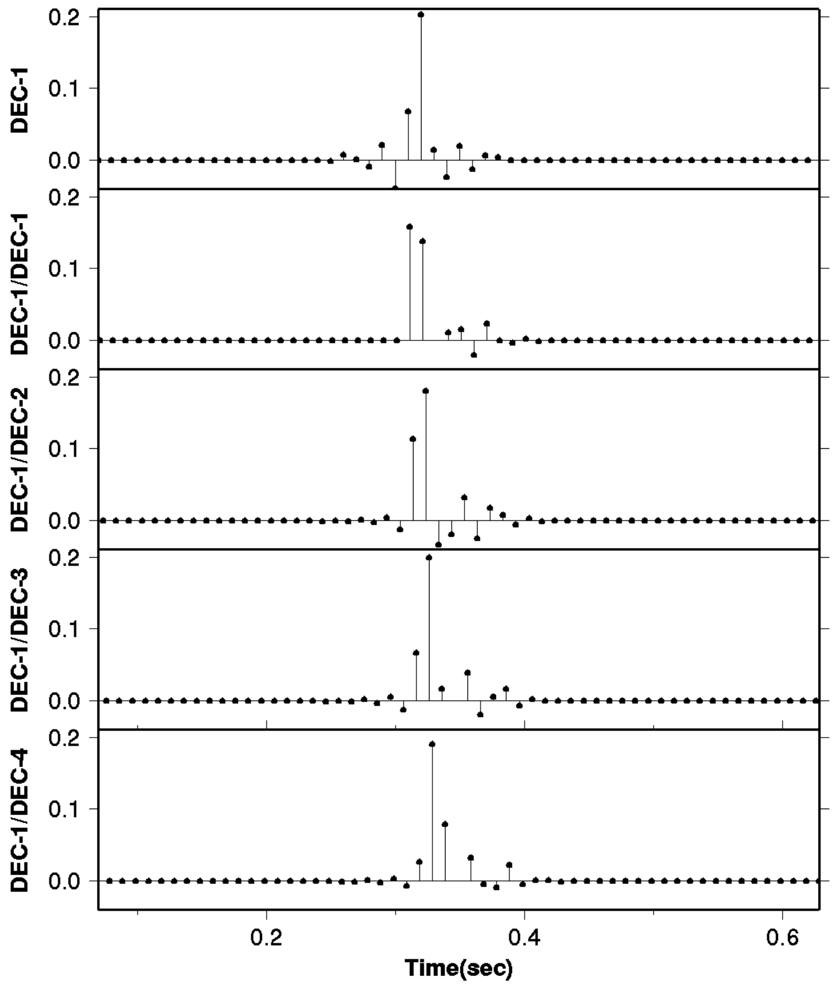
SIL FIR CORRECTION (100 Hz)

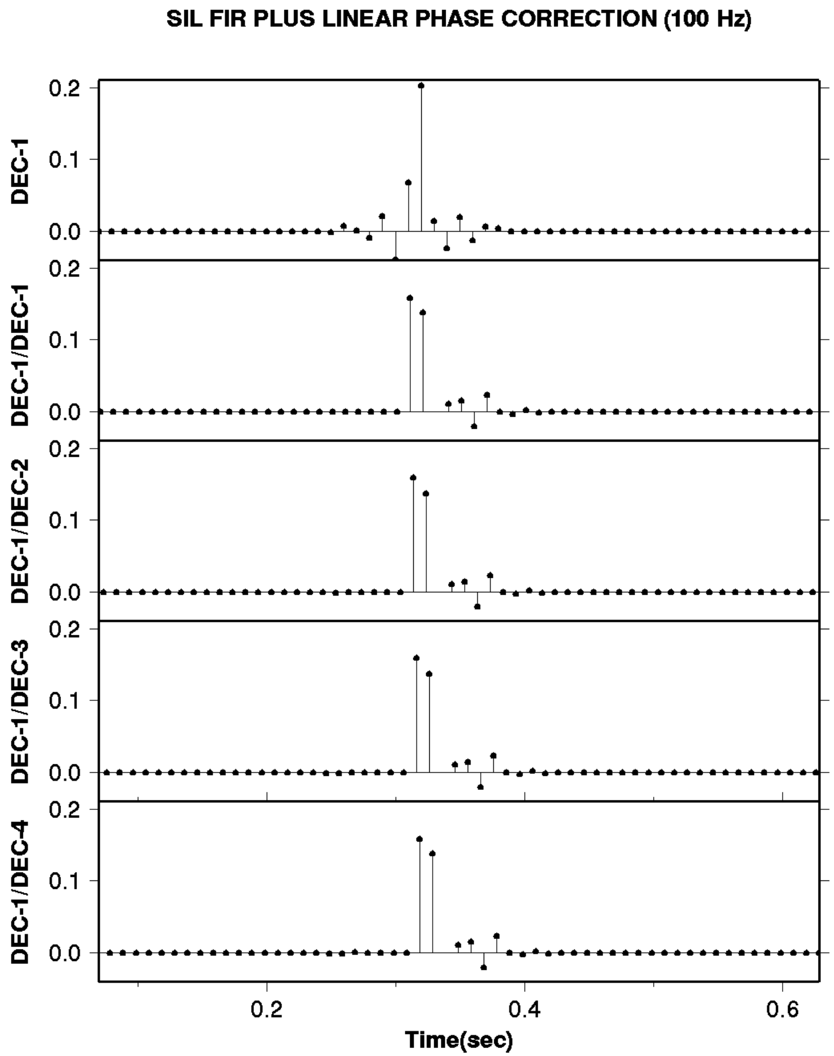


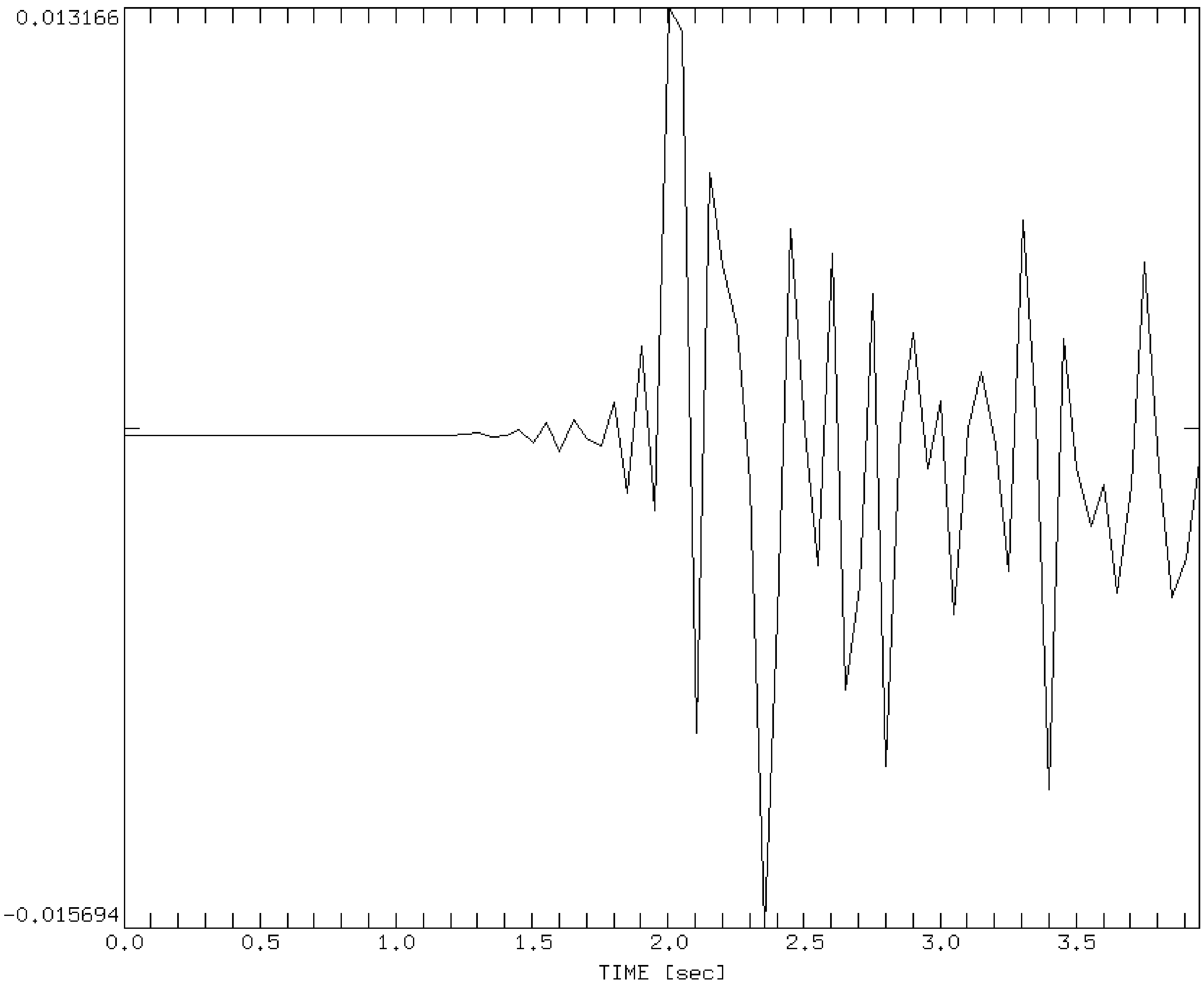


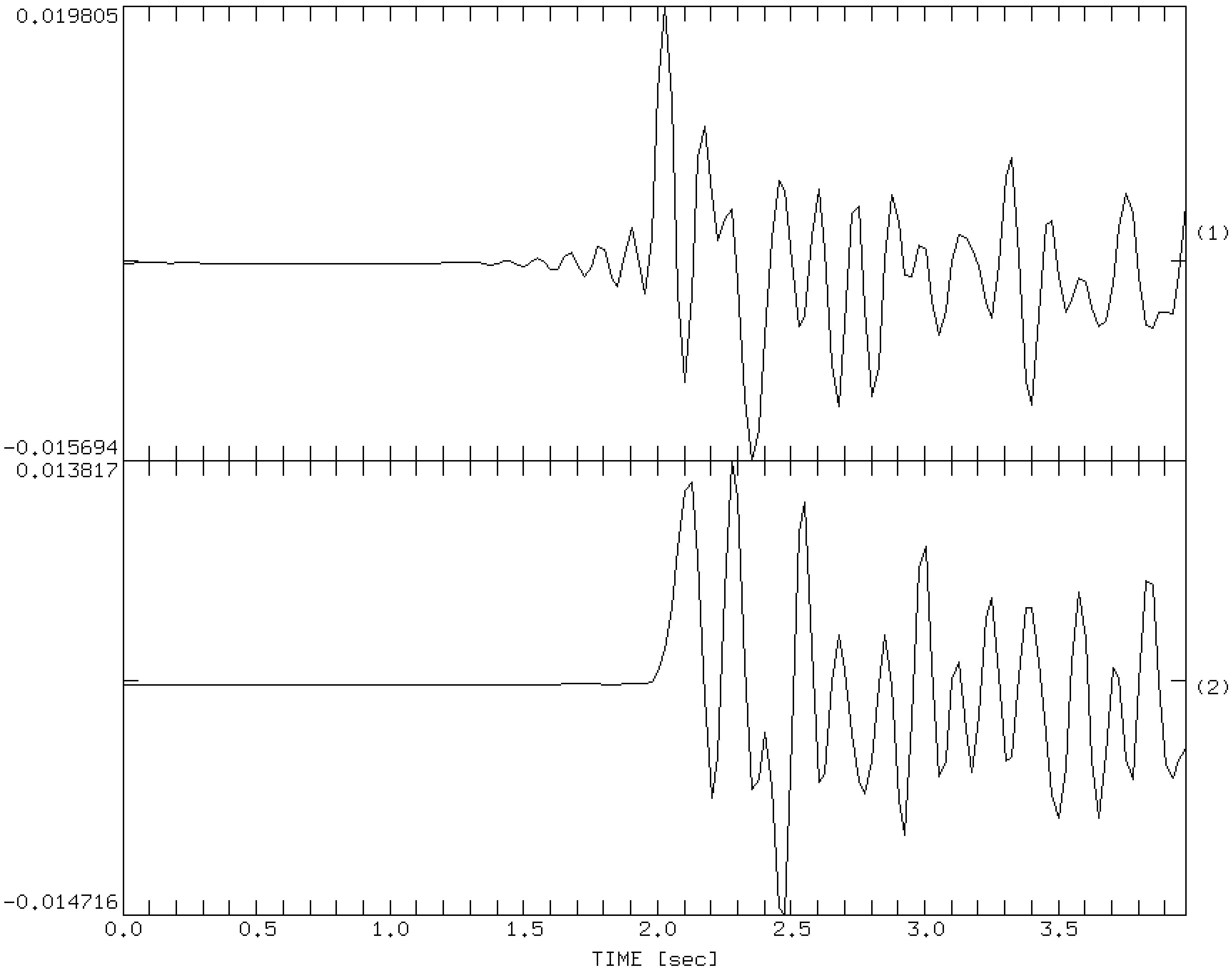
SIL SYSTEM 400/100 Hz
ORIGINAL
DEC-1
DEC-2
DEC-3
DEC-4
0.2
0.1
0.0
0.00
0.05
0.10
Time(sec)

SIL FIR CORRECTION (100 Hz)

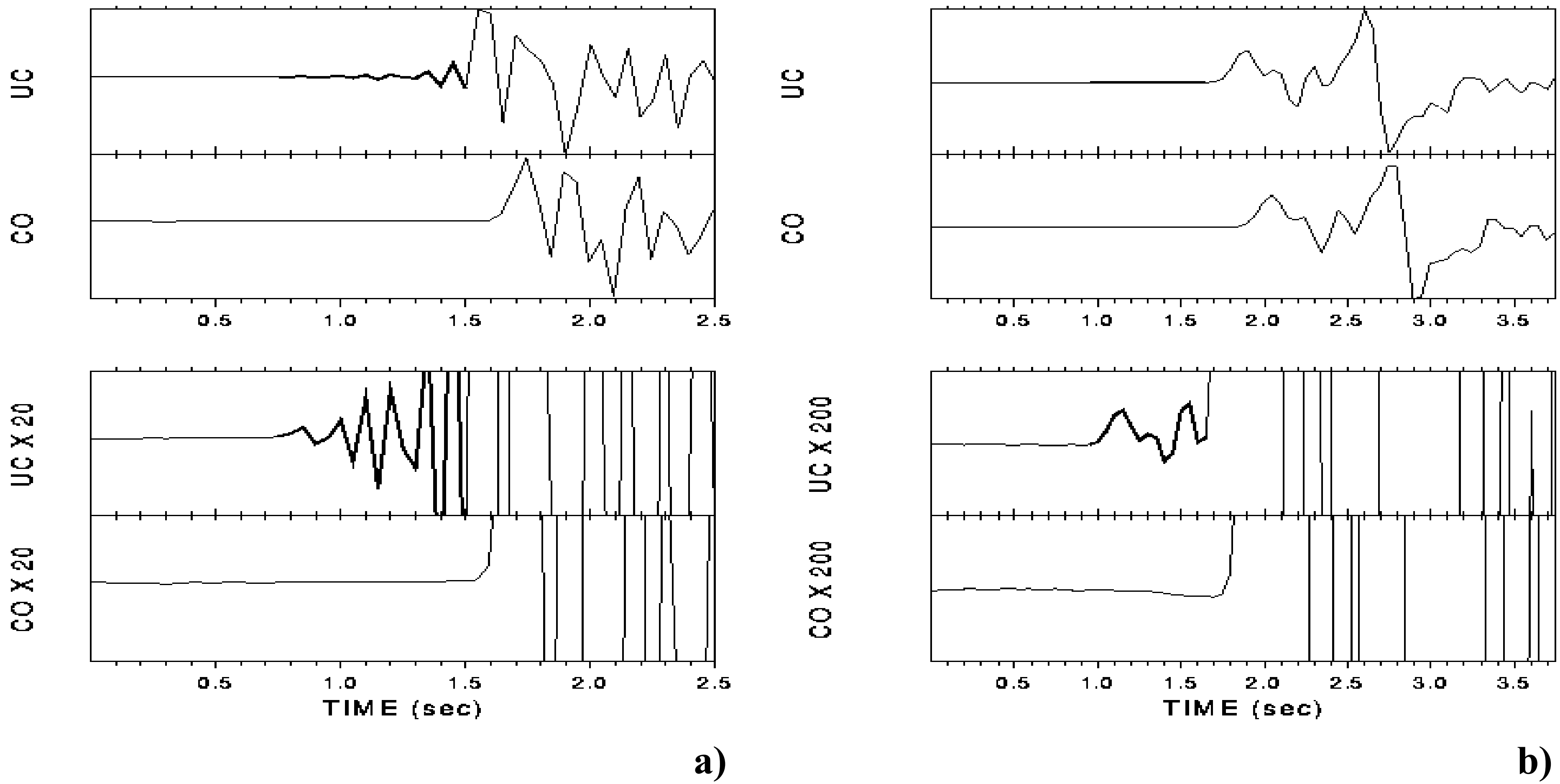



SIL FIR PLUS LINEAR PHASE CORRECTION (100 Hz)
DEC-1
DEC-1/DEC-1
DEC-1/DEC-2
DEC-1/DEC-3
DEC-1/DEC-4
0.0
0.1
0.2
0.2
0.4
0.6
Time(sec)

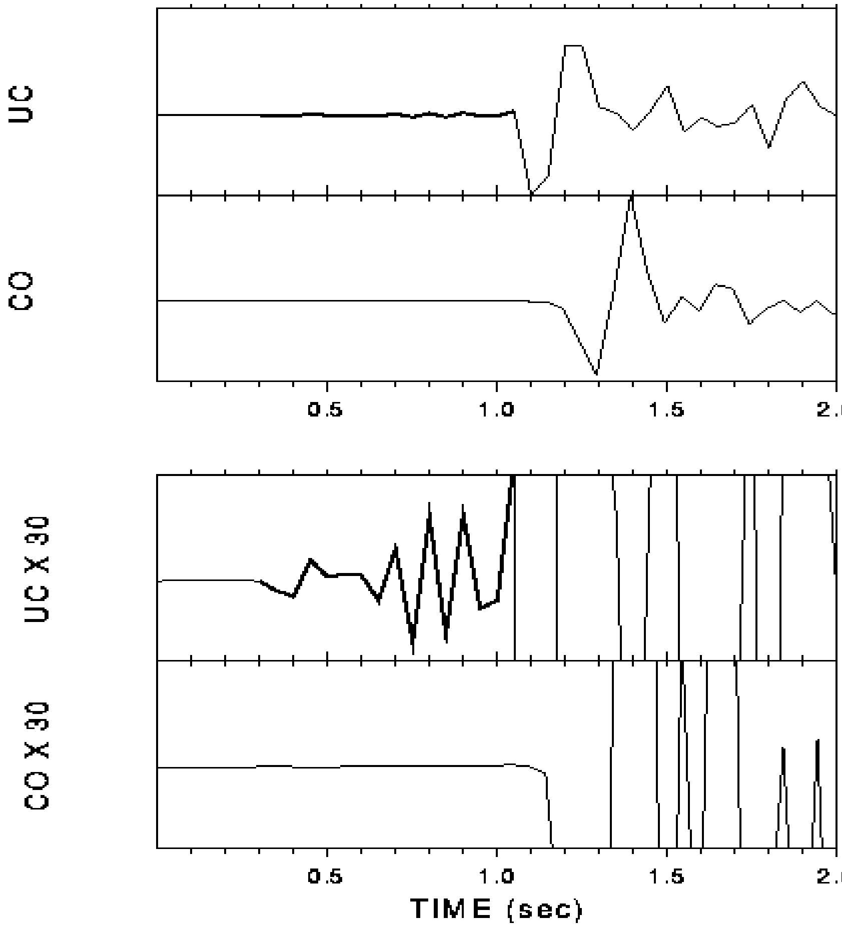

0.013166
-0.015694
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
TIME [sec]



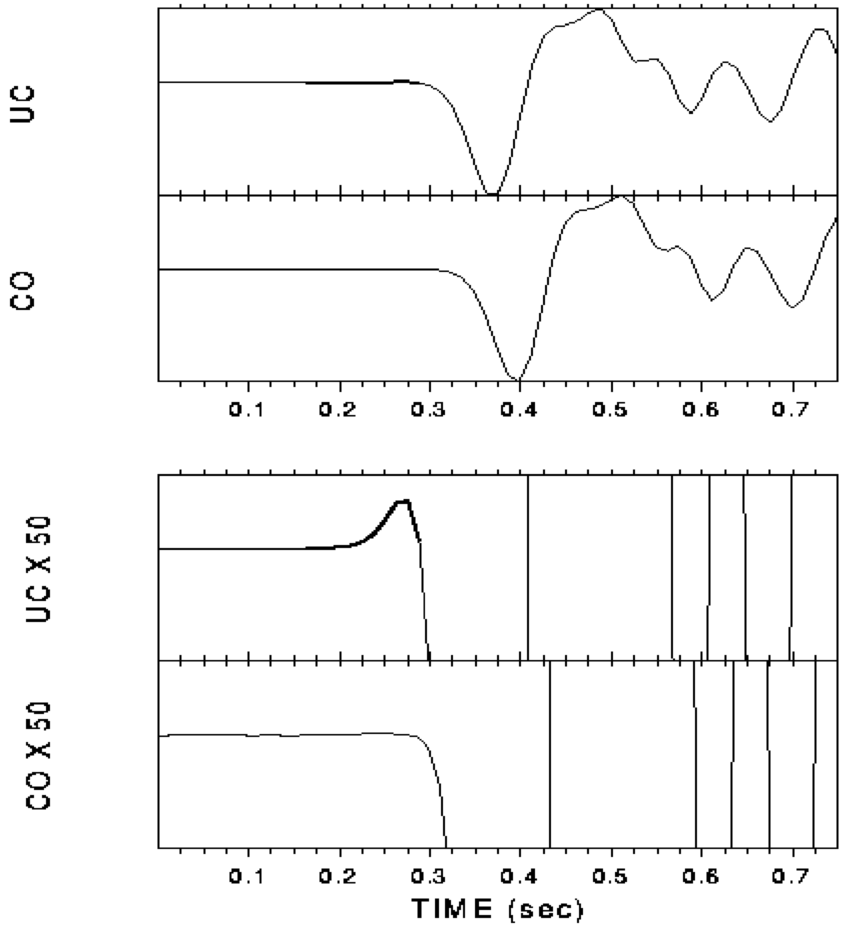
0.019805
-0.015694
(1)
0.013817
-0.014716
(2)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
TIME [sec]



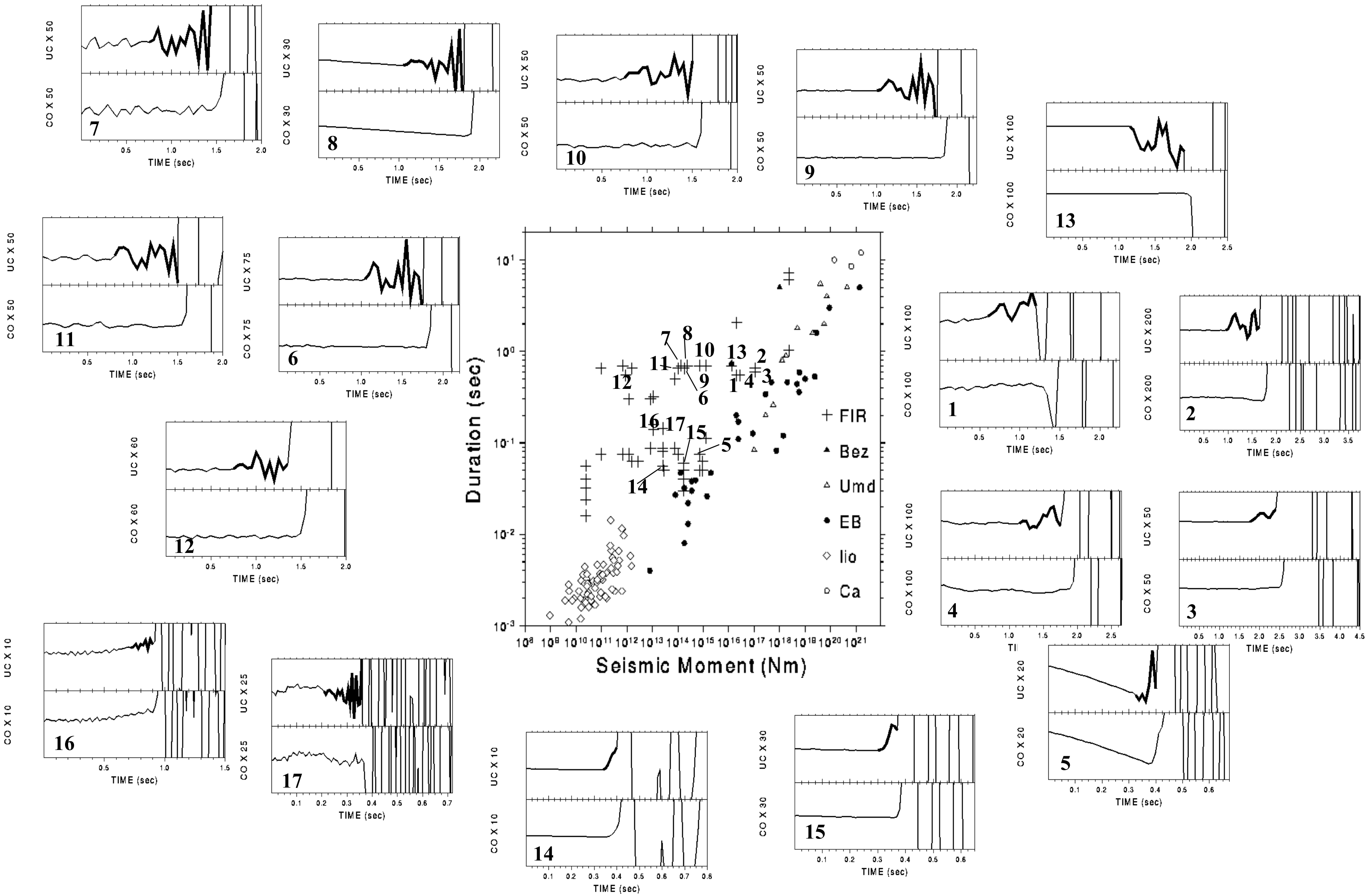
UC
CO
0.5
1.0
1.5
2.0
2.5
UC X 20
CO X 20
TIME (sec)
a)
UC
CO
0.5
1.0
1.5
2.0
2.5
3.0
3.5
UC X 200
CO X 200
TIME (sec)
b)


UC
CO
UC X 30
CO X 30
0.5
1.0
1.5
2.0
TIME (sec)

a)


UC
CO
UC X 50
CO X 50
0.1
0.2
0.3
0.4
0.5
0.6
0.7
TIME (sec)

b)


Duration (sec)
Seismic Moment (Nm)
FIR
Bez
Umd
EB
Iio
Ca
TIME (sec)
UC X 50
CO X 50
UC X 30
CO X 30
UC X 100
CO X 100
UC X 75
CO X 75
UC X 60
CO X 60
UC X 200
CO X 200
UC X 10
CO X 10
UC X 25
CO X 25
UC X 20
CO X 20
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Conclusions

FIR filter generated precursory artefacts:

- **can become impossible to be identified visually**
- **can have similar scaling properties as nucleation phases**

Zero - phase FIR filters in general

- **affect the determination of all onset properties (onset times, onset polar ities)**

Consequence

For the interpretation of onset properties (onset times, onset po larities, nucleation phases, etc.) the acausal response of the zero-phase FIR filter has to be removed

but not

for waveform analysis.