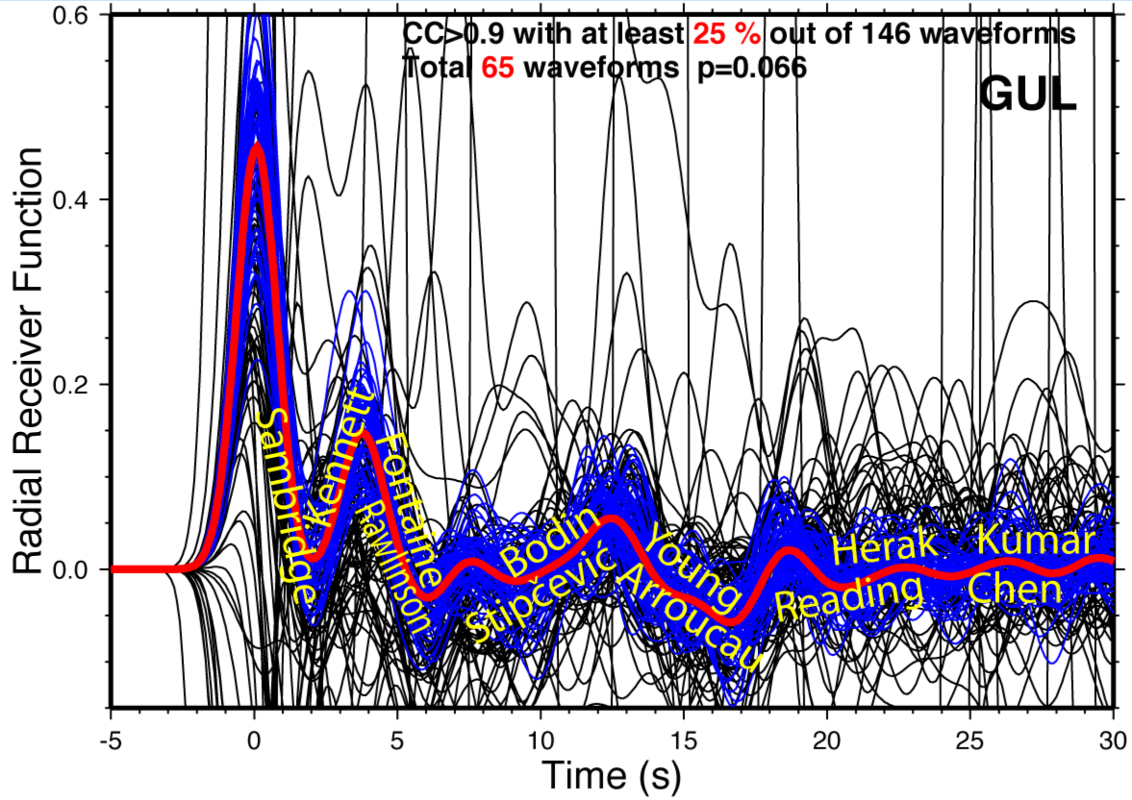
On the Limitation of Receiver-Functions Method: Beyond Conventional Assumptions & Advanced Inversion Techniques

Hrvoje Tkalčić – RSES, ANU



Acknowledgment: RSES Seismology & Mathematical Geophysics Group members

Some passive-source receiver-based methods

- Teleseismic travel times
- Teleseismic receiver functions (P and S)
- Body wave earthquake interferometry
- Ambient noise dispersion
- Interstation method surface wave dispersion

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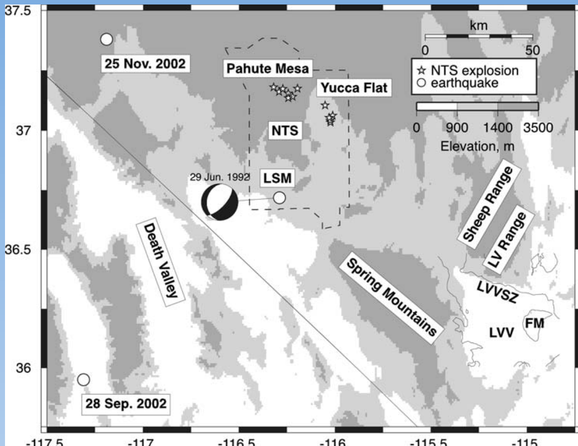
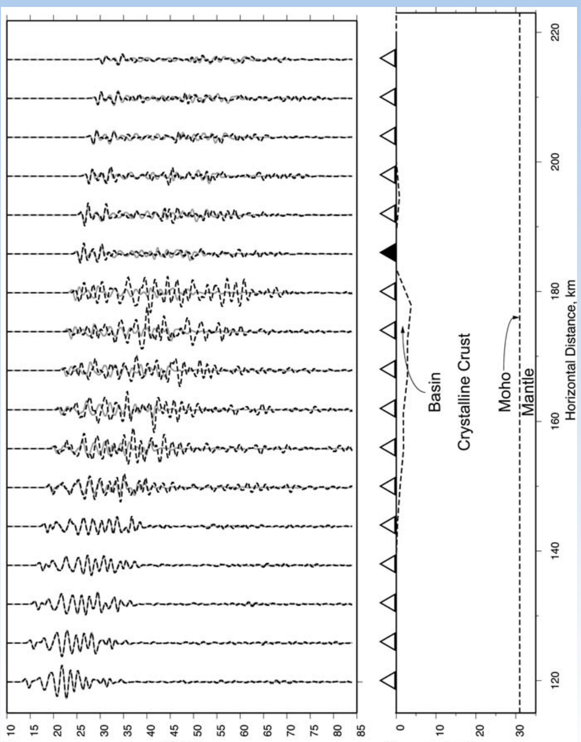
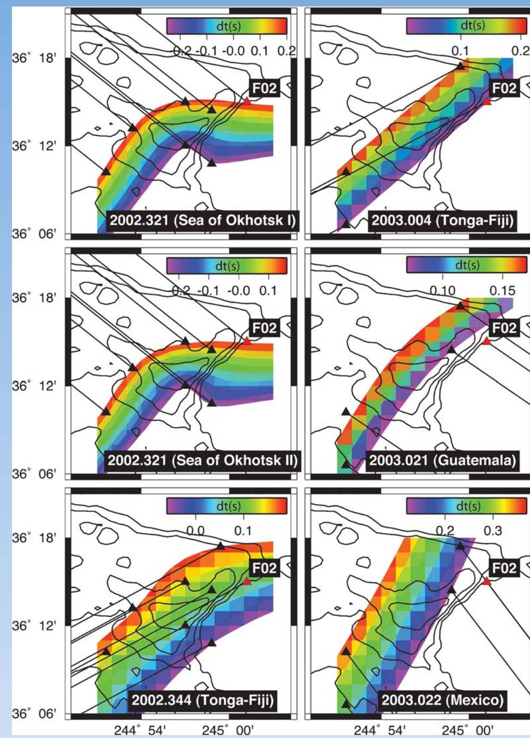
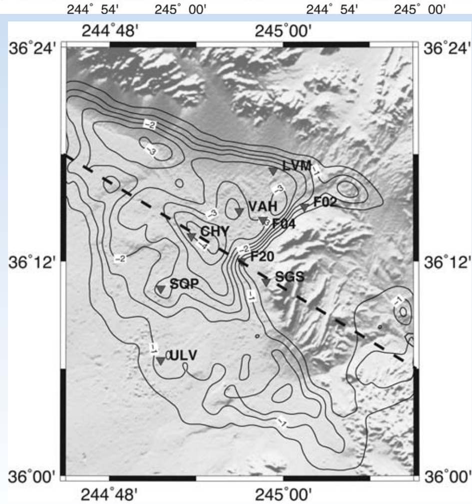
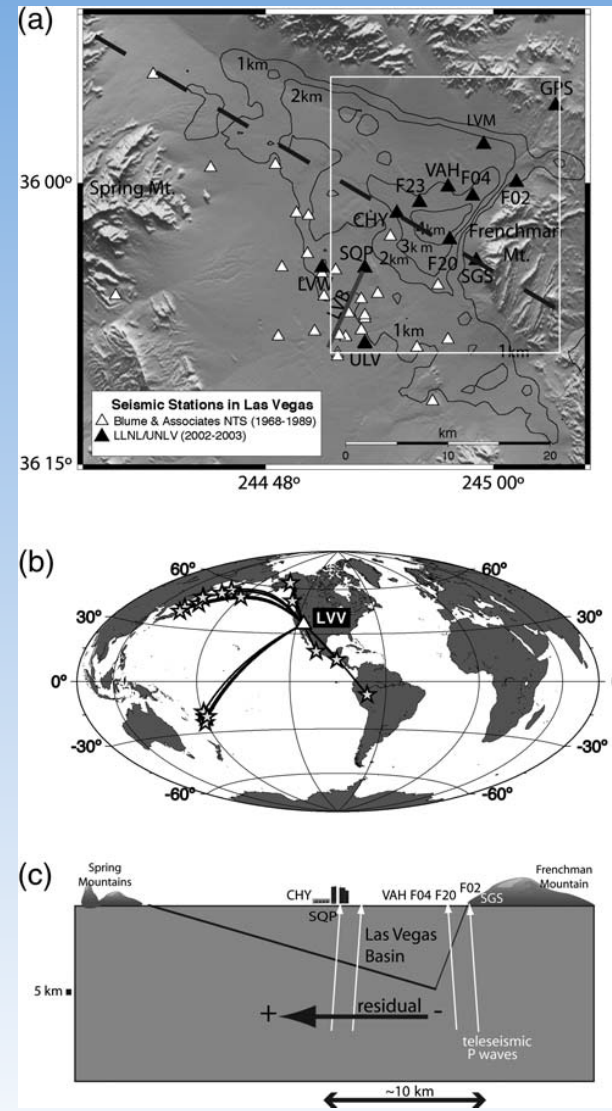
Basin structure and geometry from nuclear blasts waveforms and teleseismic travel times











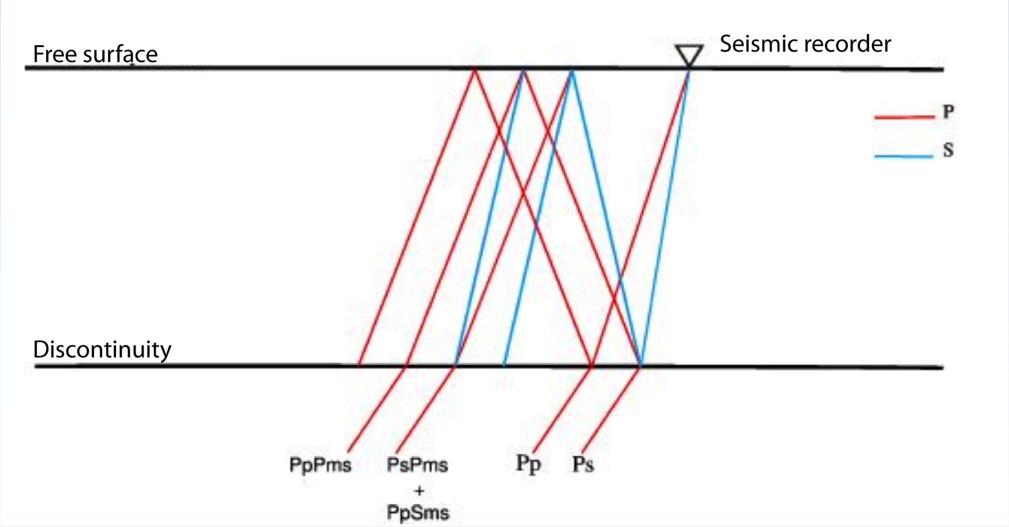
Rodgers et al. PAGEOPH 2006; Tkalčić et al., BSSA 2008

The benefits of RFs + SWD

Earth Vs structure can be inverted using:
1. Receiver functions (RF)
2. Surface wave dispersion curves
3. RF + dispersion curves (jointly) or other datasets

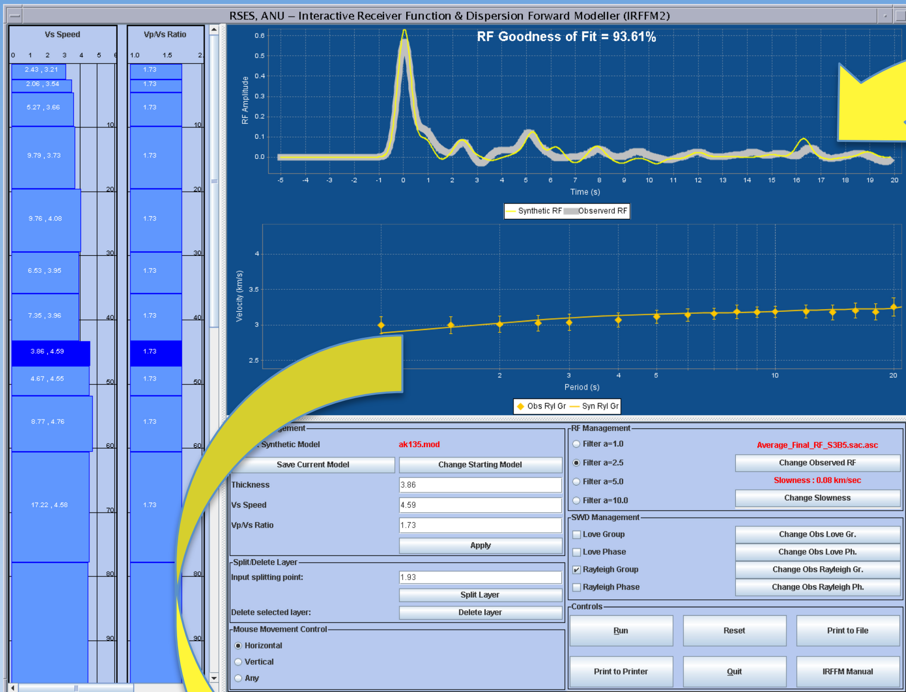
Different approaches to modeling

- Forward modeling
- Linearized inversion
- Grid-search
- Non-linear inversion with optimization
- Multi-step approach

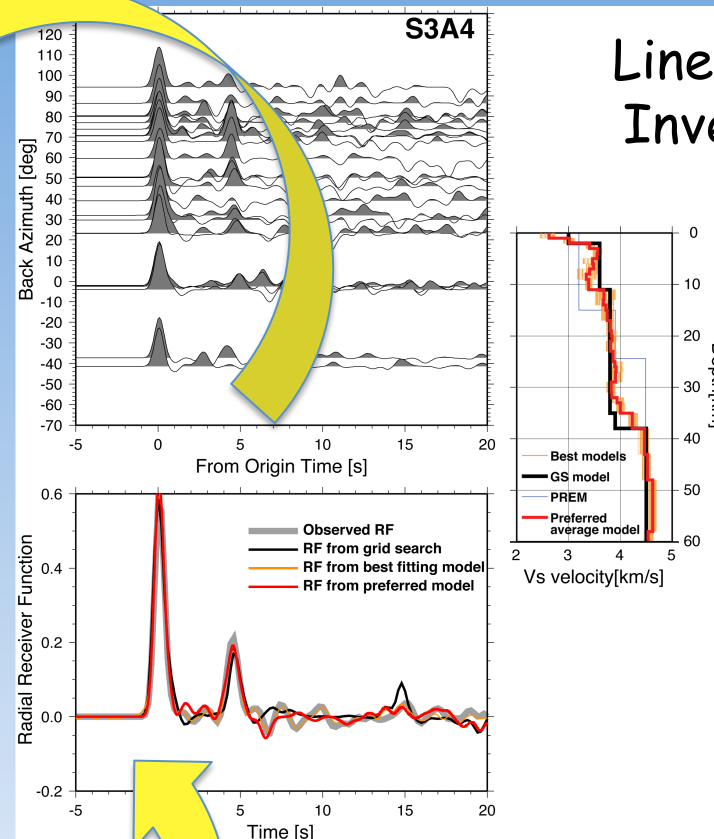


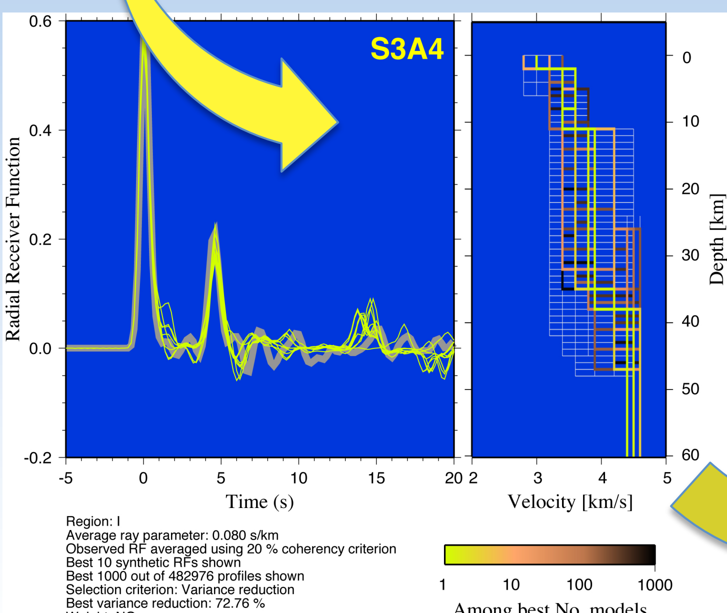
IRFFM (Interactive RF Forward Modeling)

Multi-step approach



Linearized Inversion





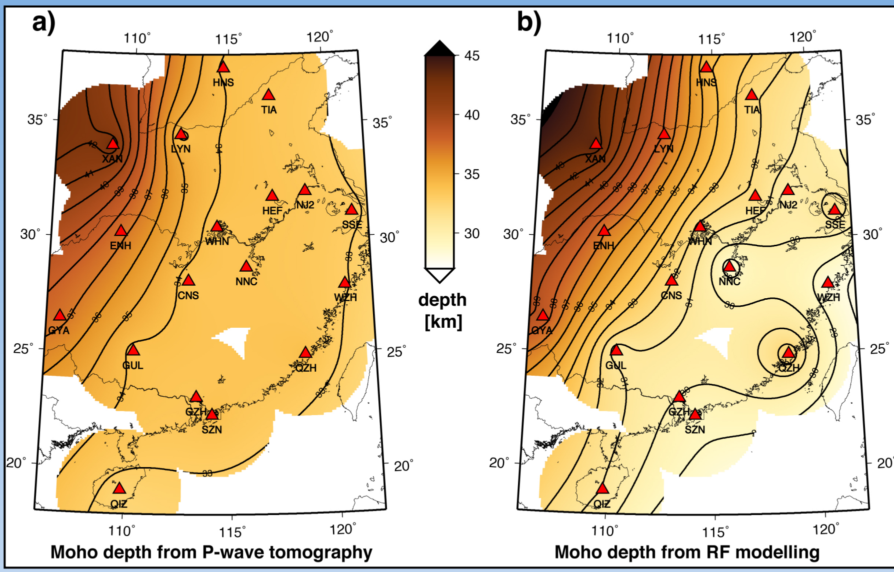
Tkalčić et al., JGR 2006

Chen et al., JGR 2010

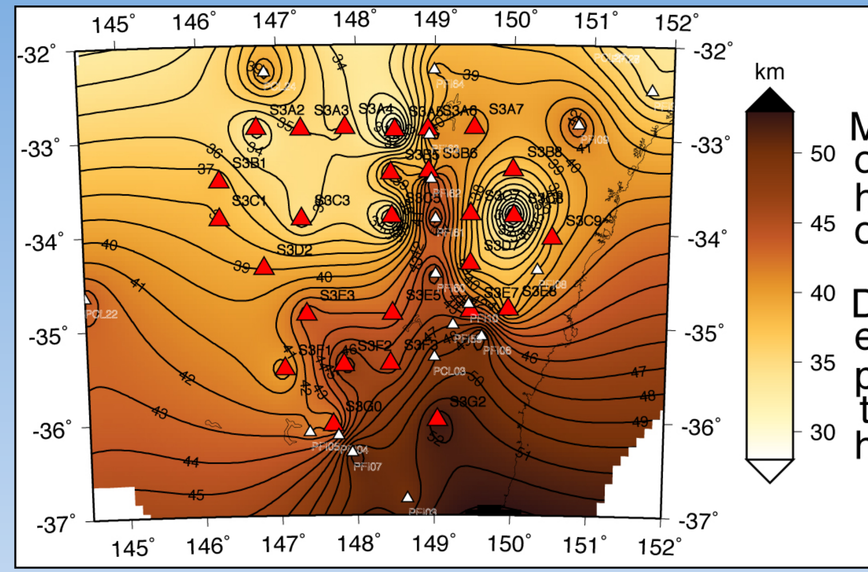
Stipčević et al., GJI 2011

Tkalčić et al., GJI 2011; GJI 2012

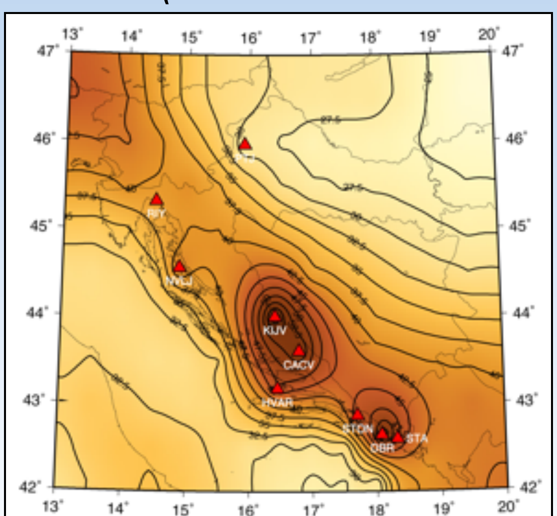
Lithospheric structure of Saudi Arabia, China, Australia & Croatia from multi-step modeling of RFs and SWs



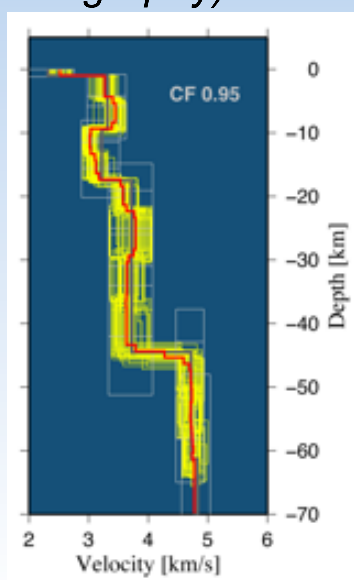
SE China (RFs combined with tomography)



SE Australia (RFs combined with ambient noise)



Croatia and Adriatic Sea



Advantages and limitations of RFs

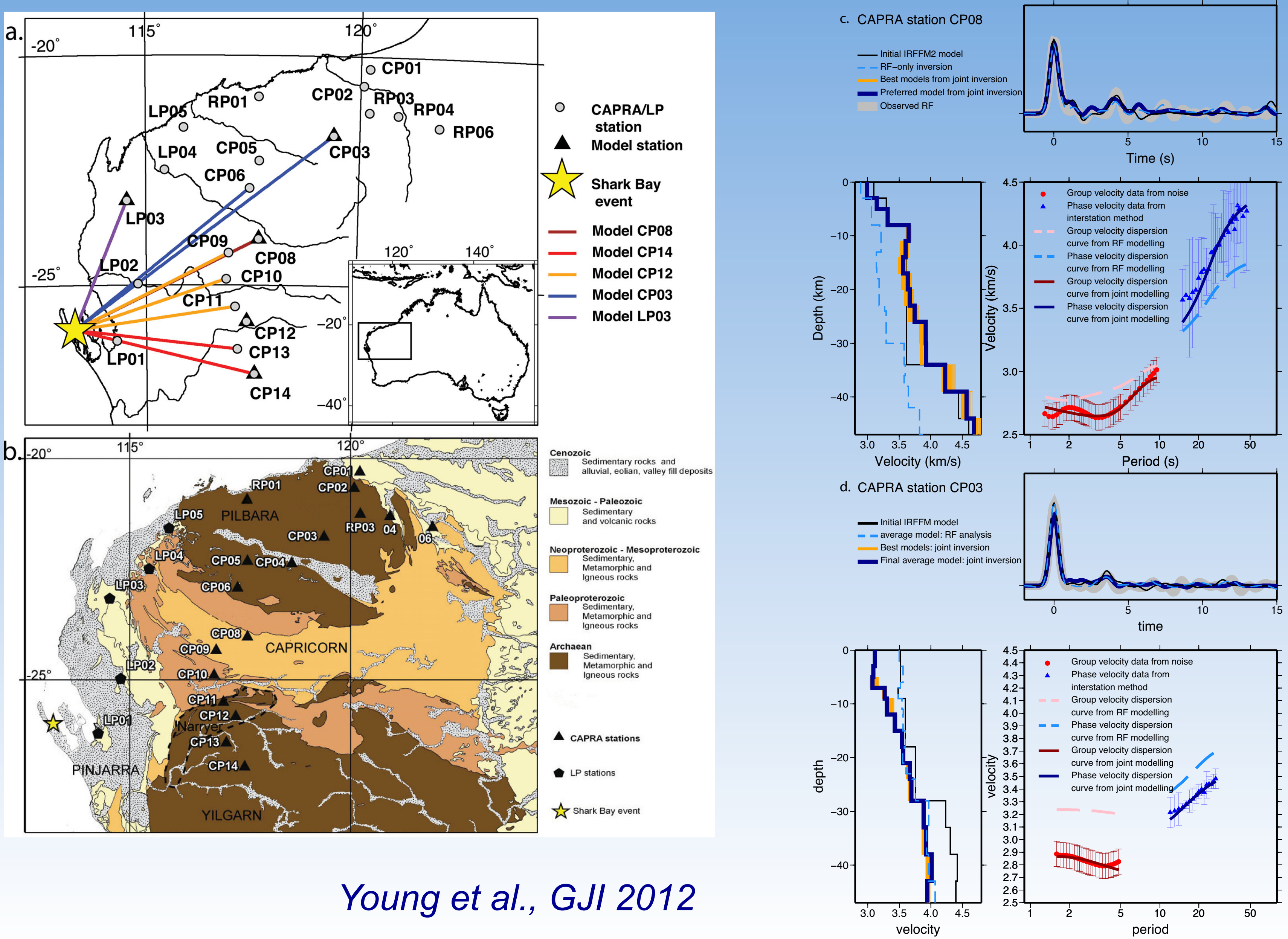
Advantages

- A way to invert for Vs structure under a single station
- Sensitive to gradients (discontinuities) in Vs velocity
- A needed complement to crustal tomography
- RF + SW dispersion curves (jointly) or other datasets

Limitations of conventional methods

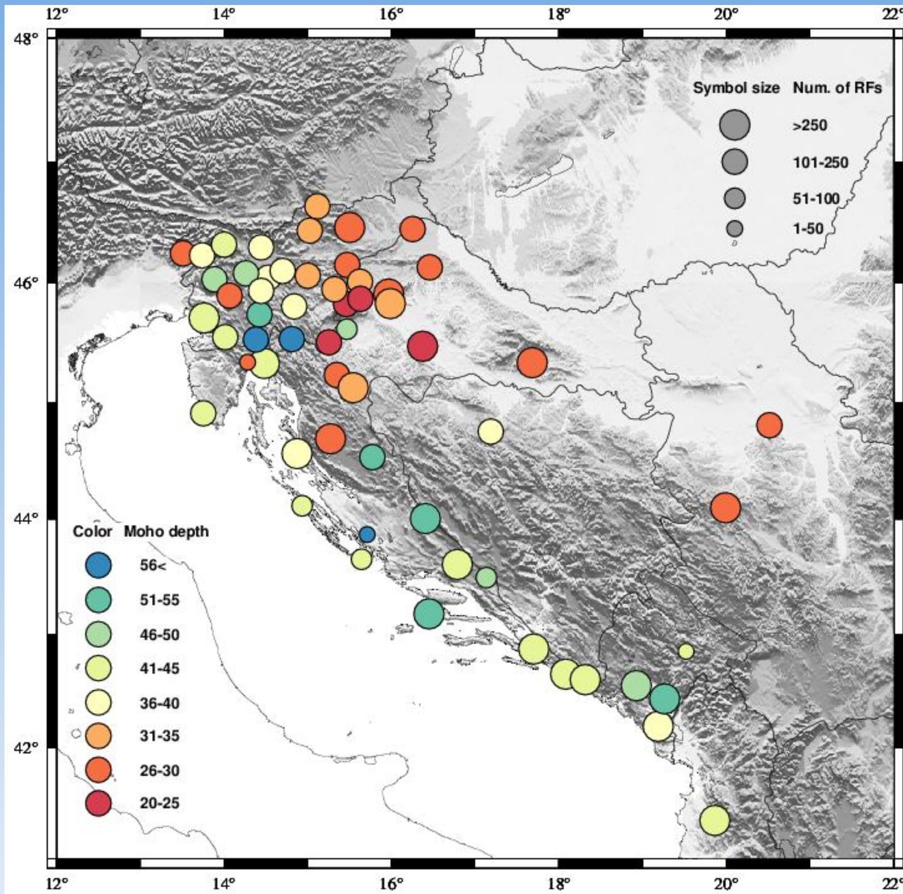
- Information limited to a volume beneath a single station
- Insensitive to absolute velocity unless SW are added
- Simplifications/assumptions often cannot explain real Earth (1. lack of data, 2. anisotropy, 3. dipping layers, 4. non-uniqueness and noise in the data)

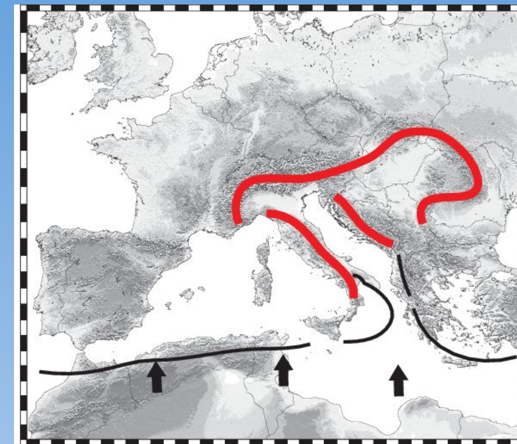
1. Exploiting seismic signal and noise in an aseismic environment to constrain crustal structure



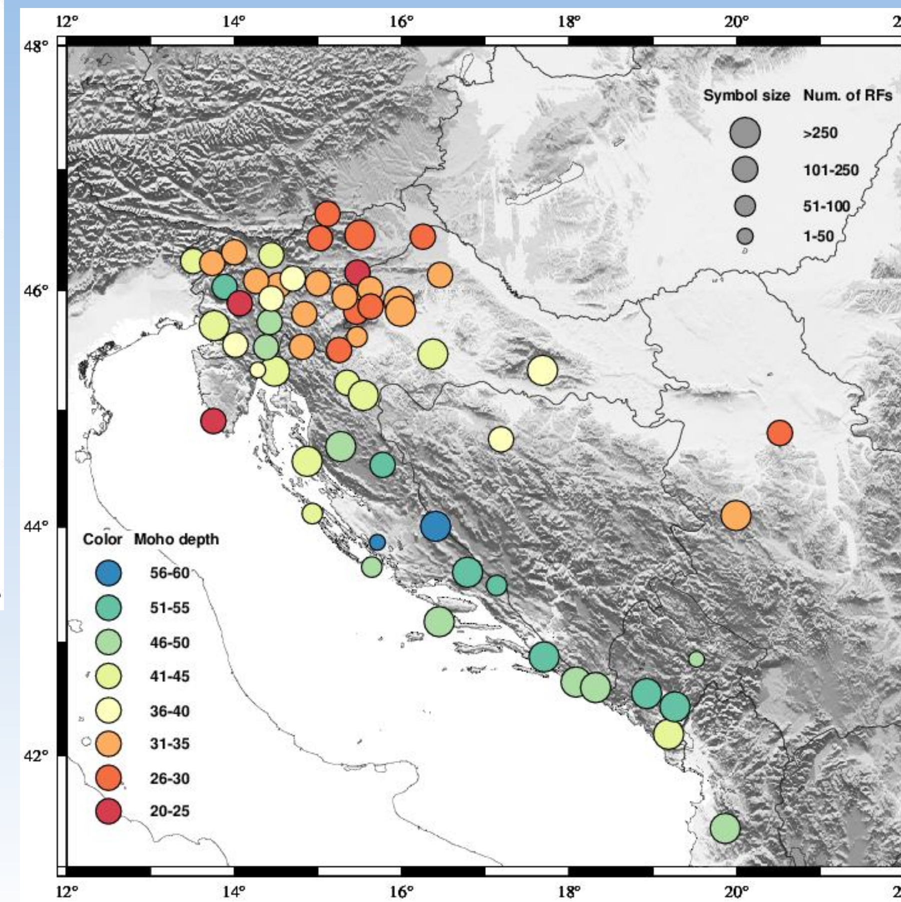
Young et al., GJI 2012

2. Dipping Moho

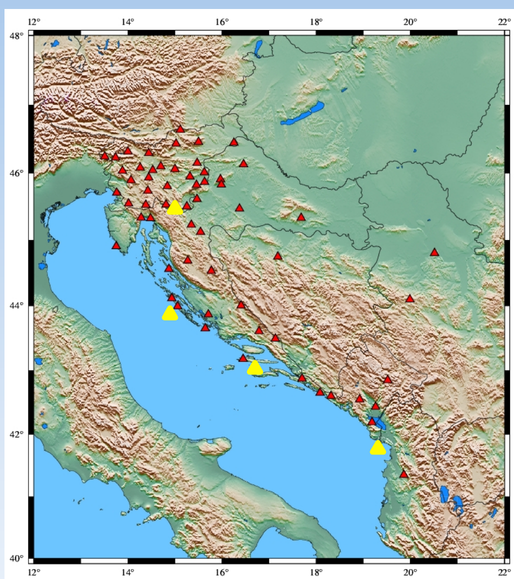


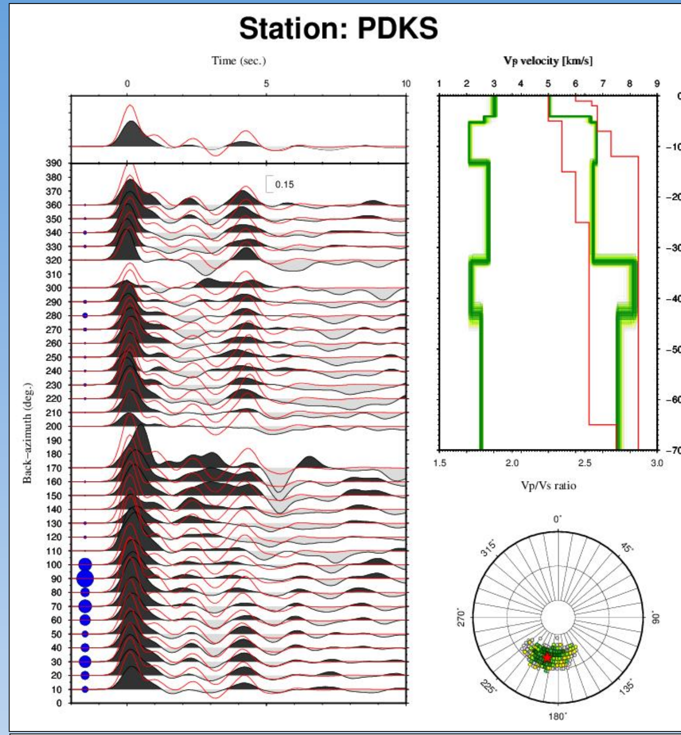


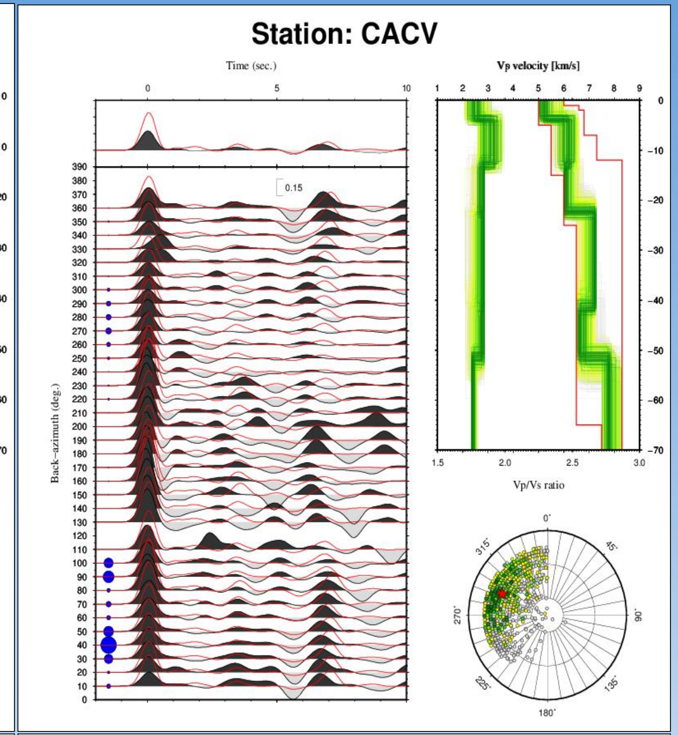
From Stipčević,
PhD Thesis,
paper in preparation

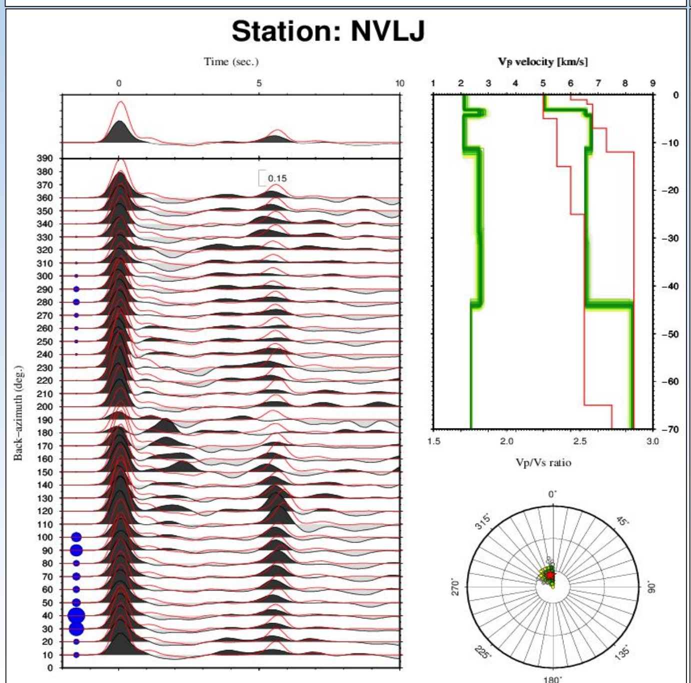


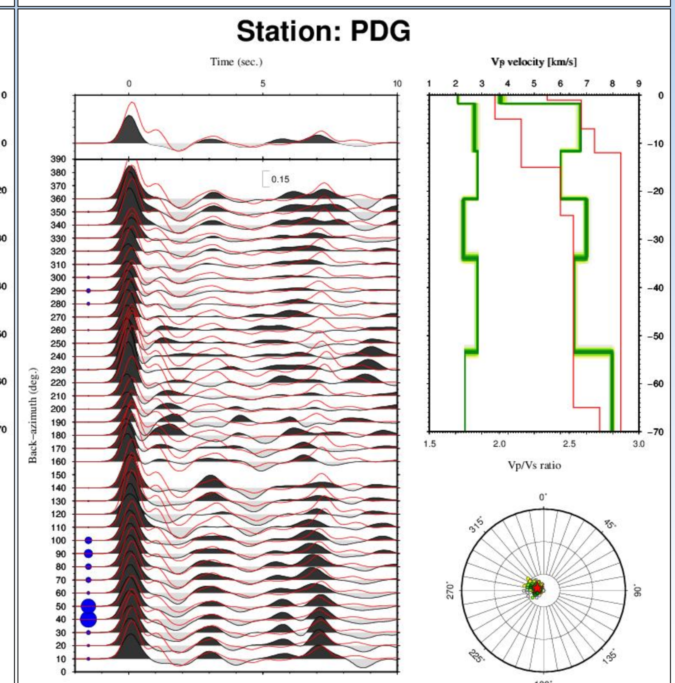
Moho depth determined using H–κ (above) and NA (right) method



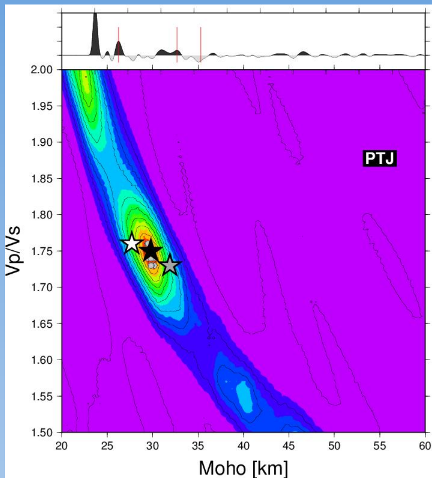


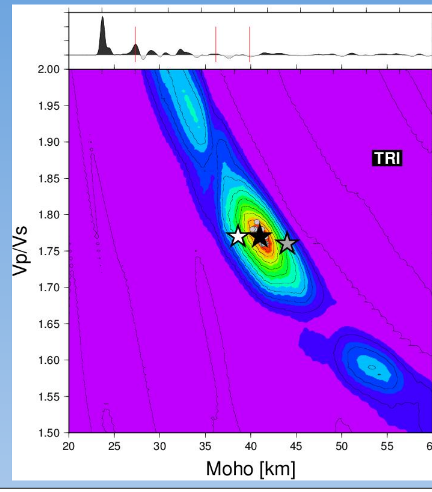


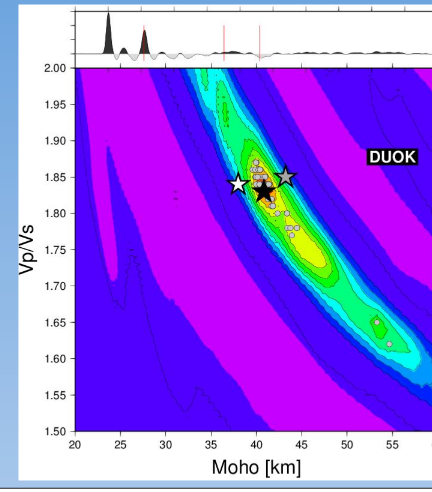


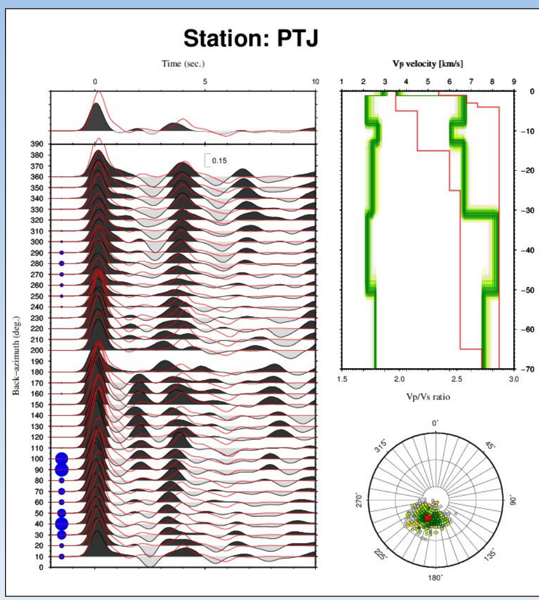


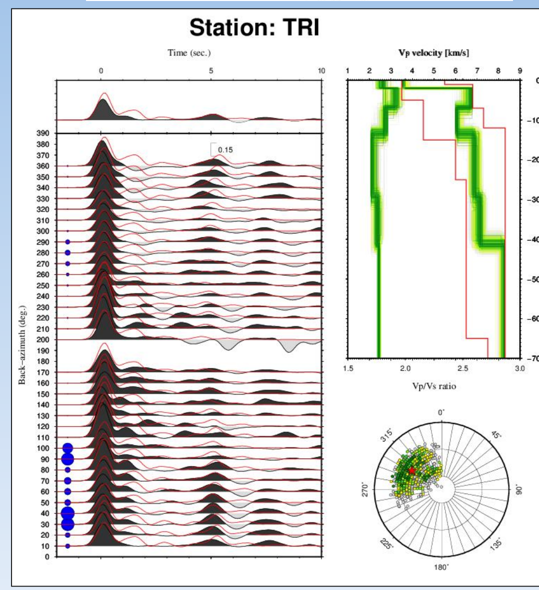
*From Stip*čević,
PhD Thesis,
paper in preparation

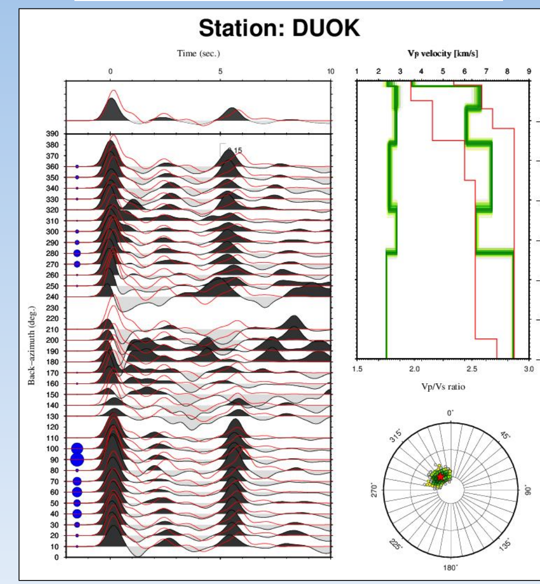






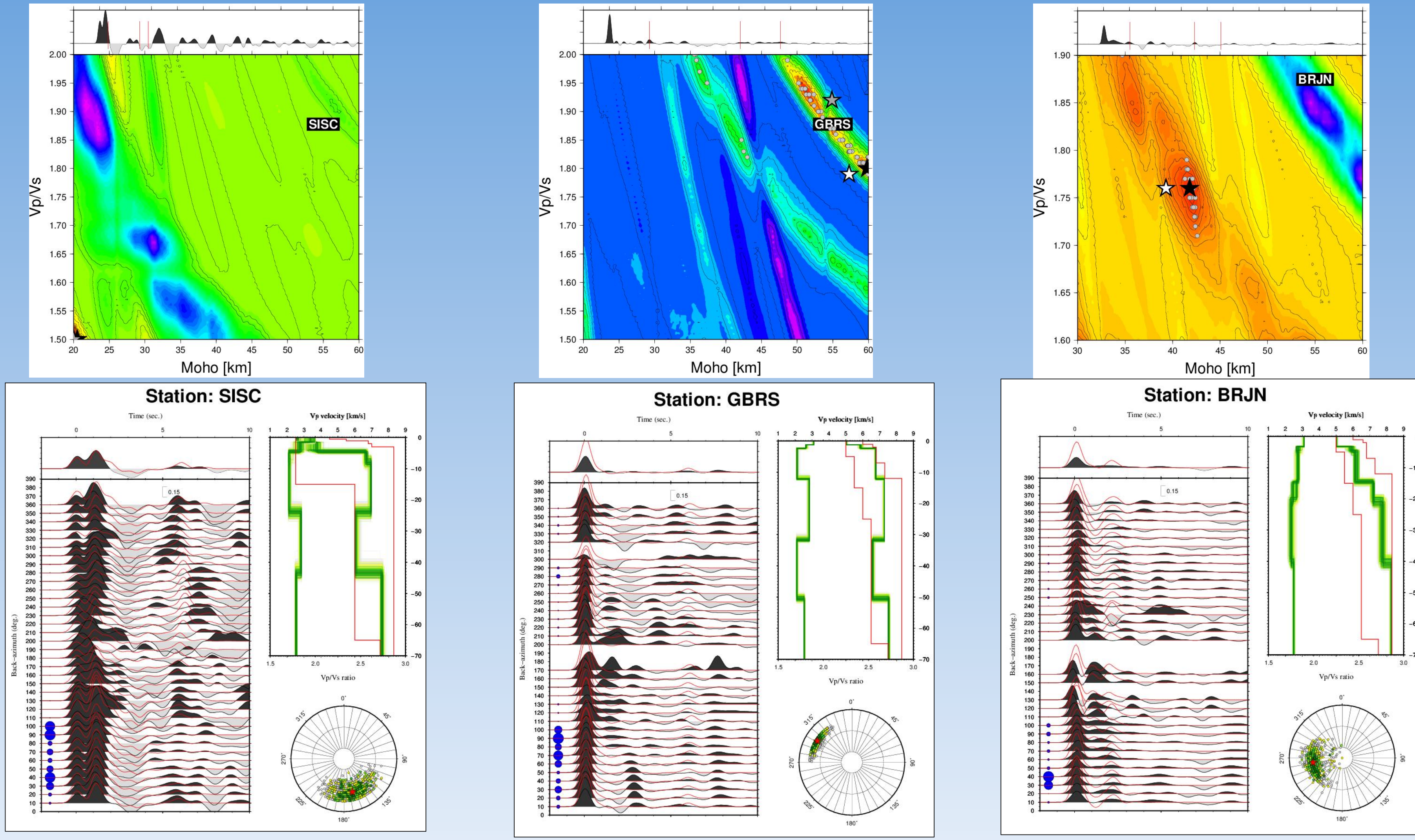






Stations with similar results obtained using H-κ and NA methods

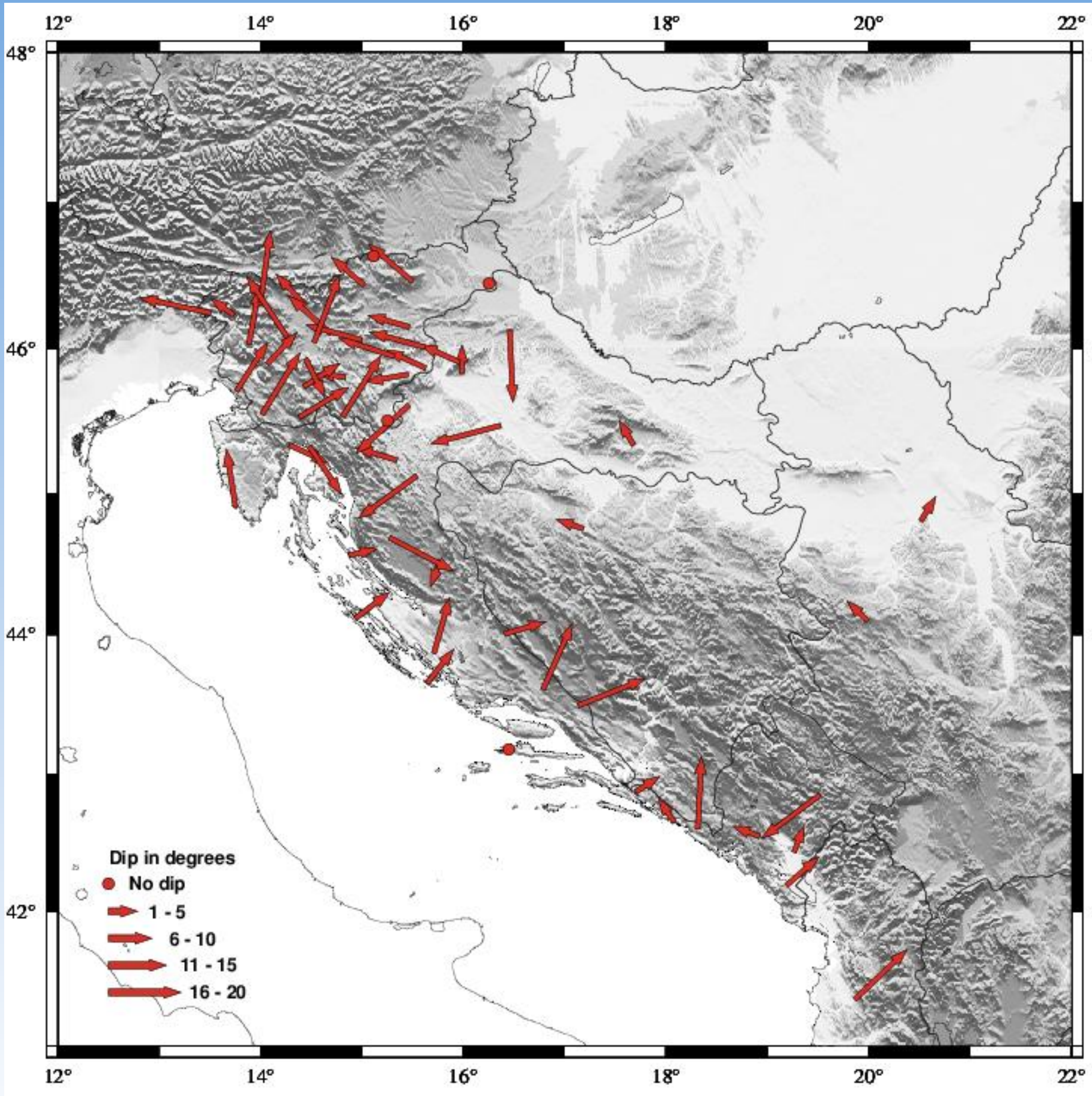
*Stip*čević et al., in preparation



Stations for which there is a large difference between the H-κ i NA results

*Stip*čević et al., in preparation

Moho dip determined using NA algorithm



*Stip*čević et al., in preparation

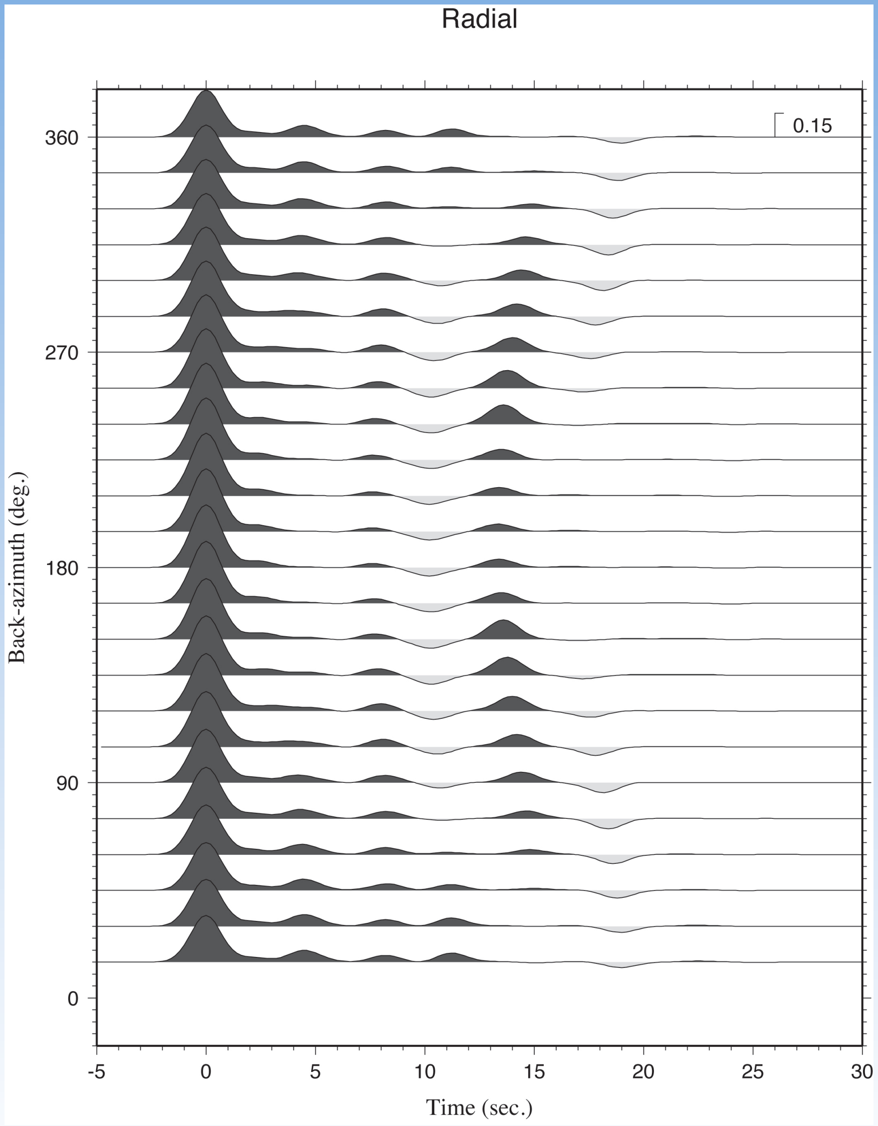
Dipping Moho - synthetic experiment

<u>Starting model :</u>

Two 20km thick layers in the crust
Moho at 40 km
with 20° dip & 270° strike

Synthetics are calculated using Fredrickson and Bostock method assuming an isotropic medium, and synthetic RFs are determined by deconvolution.

These synthetic RF data are then linearly stacked and inverted for Earth structure using NA method introduced in Exercise 6.



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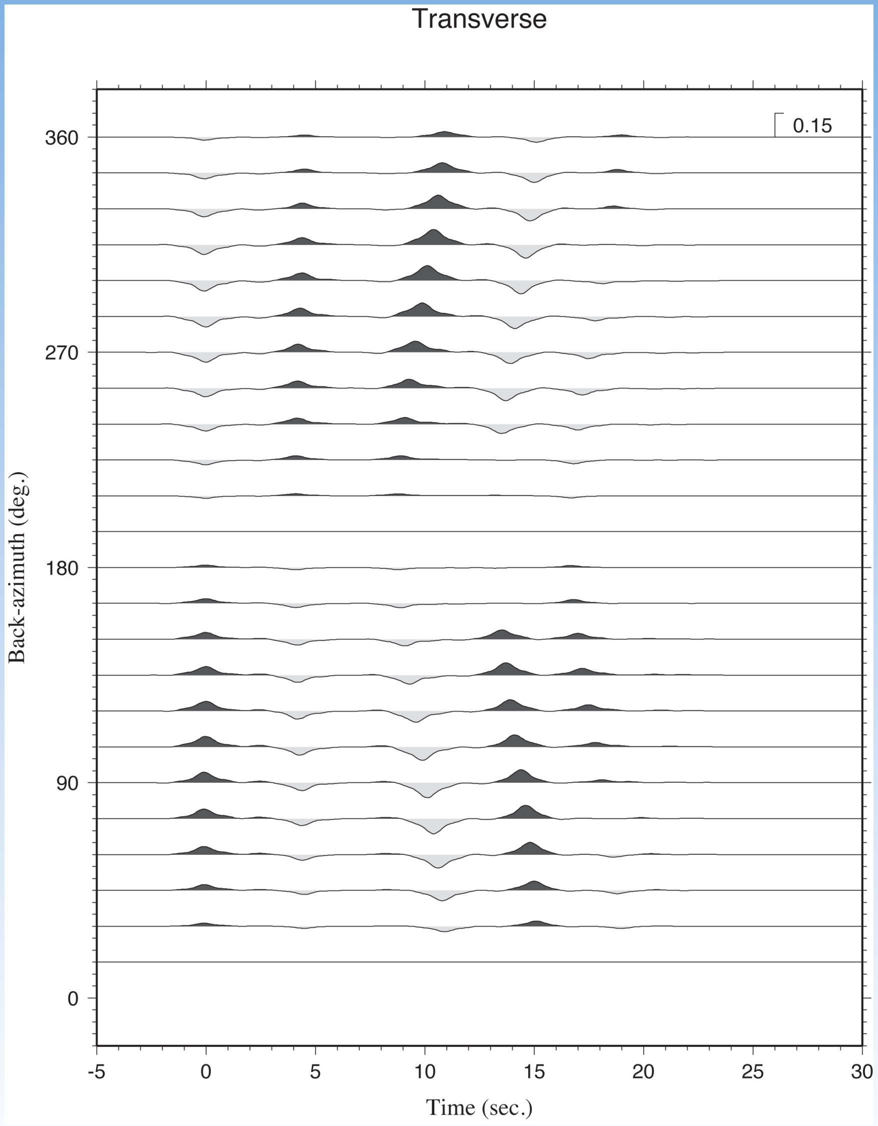
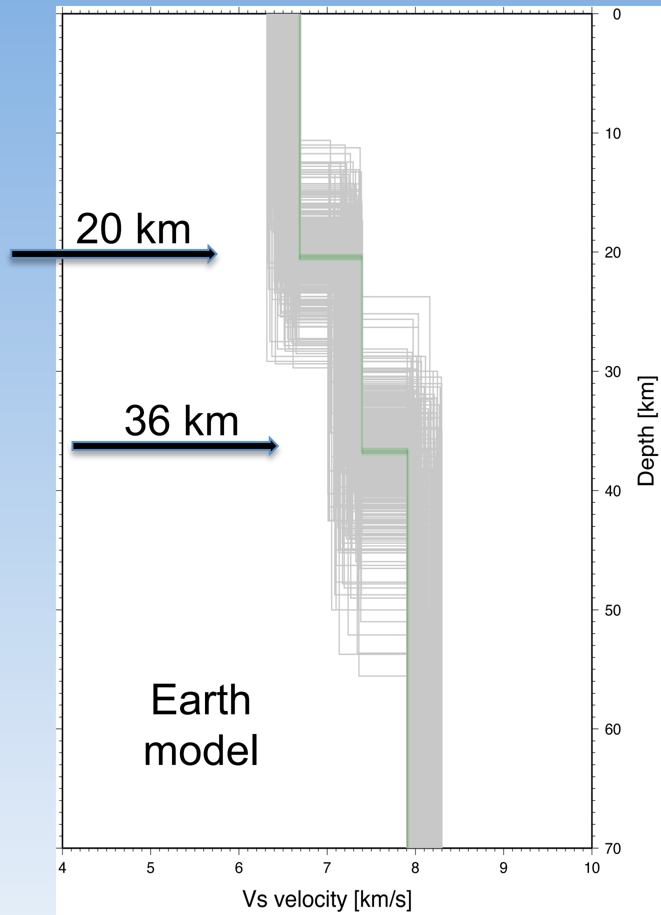


Dipping Moho - Synthetic Experiment

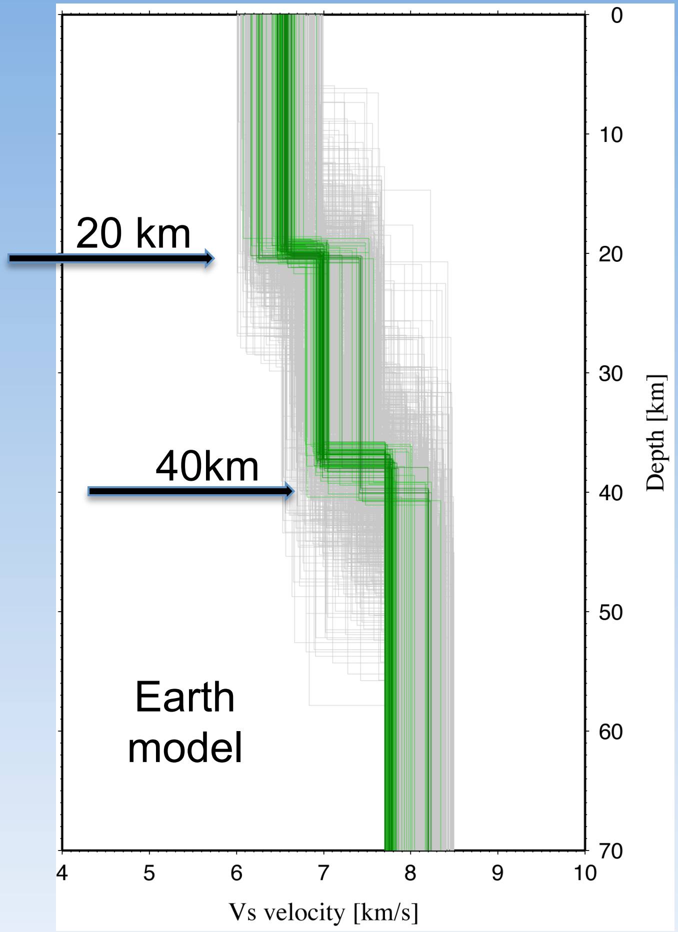


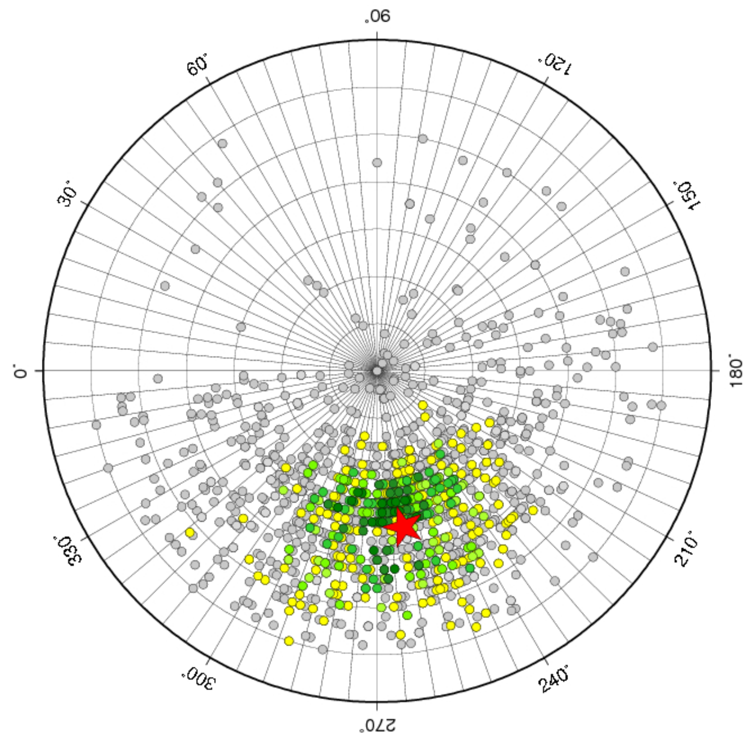
This is a result of the NA inversion when laterally homogeneous horizontal layers are assumed.

It is also assumed that the Moho is horizontal (not dipping).

(Exercise 10 in synopsis)

Dipping Moho – Synthetic Experiment

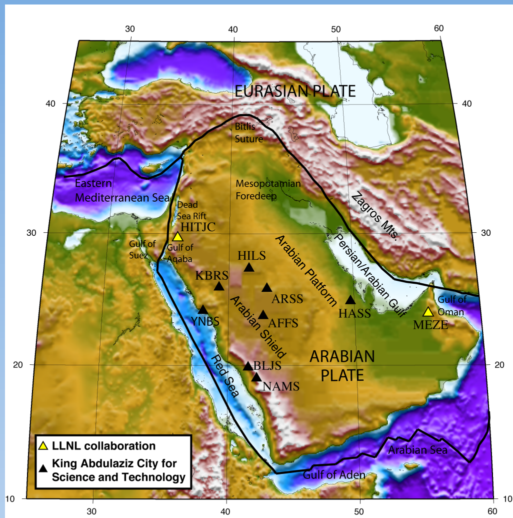


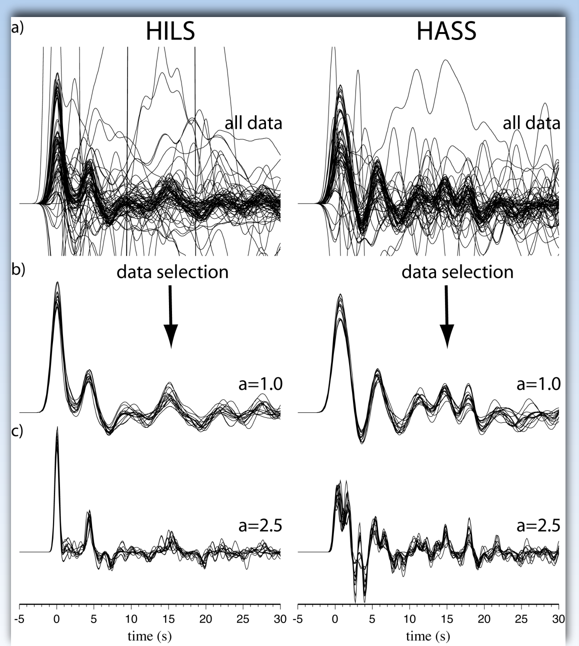


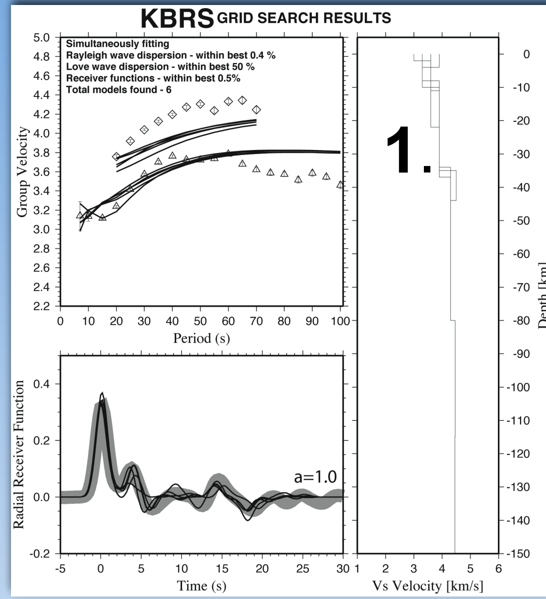
Moho: D~17°; S~262°

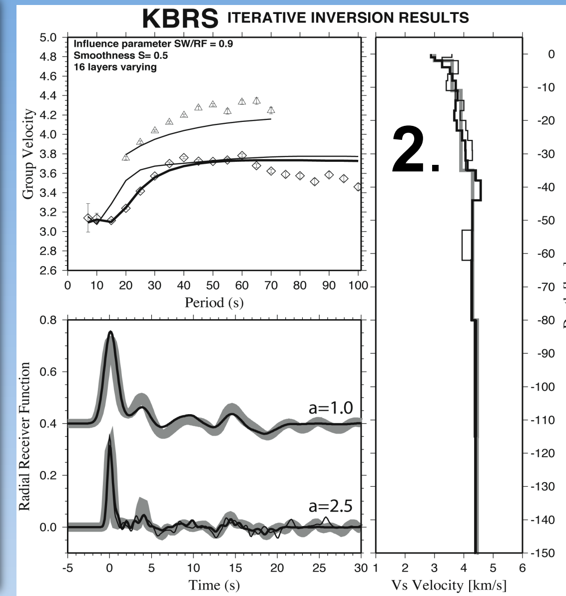
Now inverting for the Moho dip and orientation

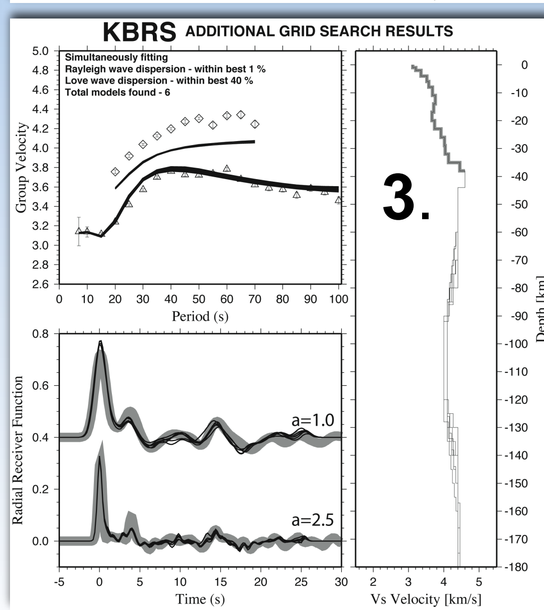
3. A multi-step approach including polarization anisotropy

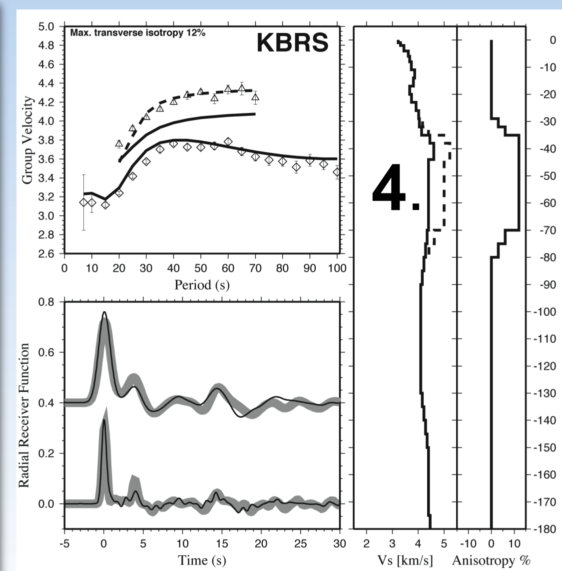












Tkalčić et al., JGR 2006

4. Non-uniqueness etc.

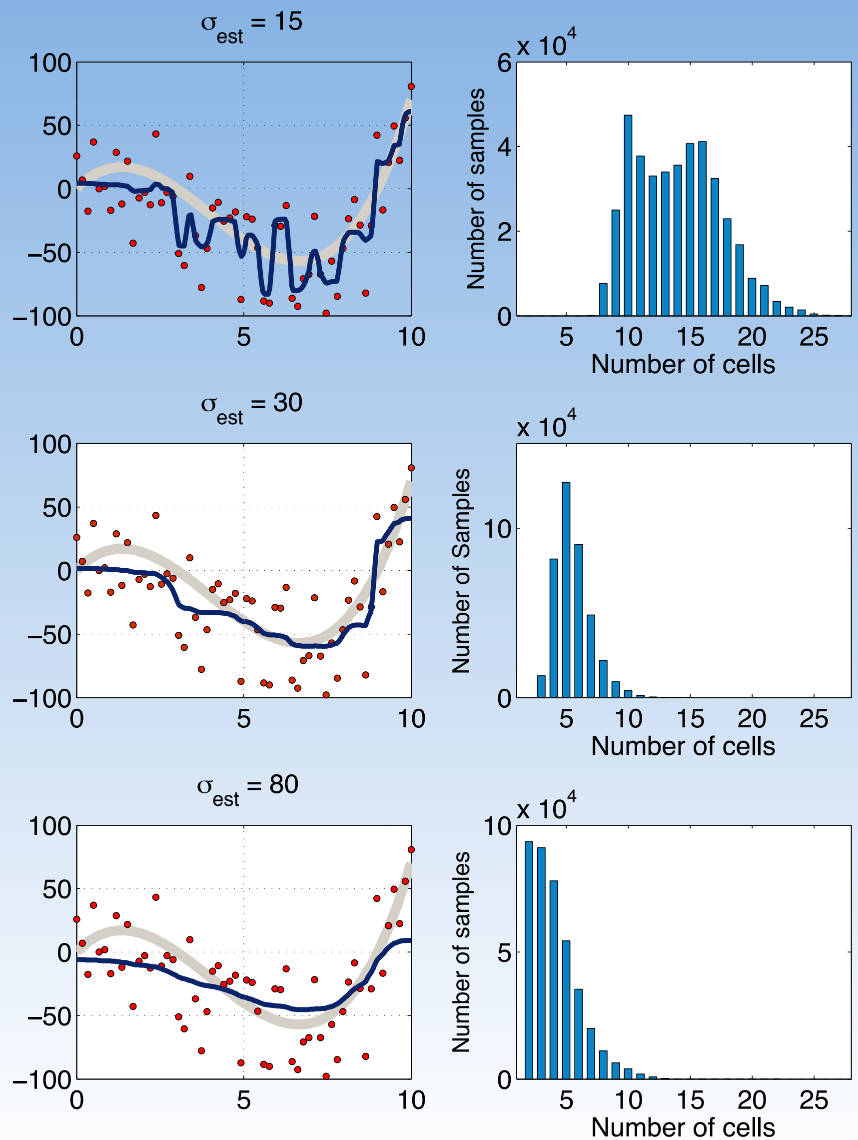
Different approaches to inverse problems

- Forward modeling
- Linearized inversion
- Grid-search
- Non-linear inversion with optimization
- Multi-step approach
- Non-linear inversion with the Bayesian framework
- Transdimensional Bayes framework…hierarchical

Bayes theorem:

$$p(m \mid d_{obs}) \propto p(d_{obs} \mid m)\, p(m)$$

The importance of knowing the data noise in trans-dimensional formulation



Hierarchical Models

- Relationship between data noise and model complexity
- Treating data noise σ as an unknown in the problem

Data noise is uncorrelated

Likelihood function

unknown

$$p(d \mid m) \propto \frac{1}{\sqrt{\left(2\pi\sigma^2\right)^N}} \exp\left[\frac{-\left\| d - g(m) \right\|^2}{2\sigma^2}\right]$$

unknown

unknown

Data noise = measurement uncertainty + modeling uncertainty

Covariance Matrix of Noise in Data

Data noise is correlated

Misfit $$\Phi(m) = [d - g(m)]^T C_D^{-1} [d - g(m)]$$

Likelihood $$p(m \mid d) = \frac{1}{\sqrt{(2\pi)^N |C_D|}} \exp\left[\frac{-\Phi(m)}{2}\right]$$

Noise Parameterization

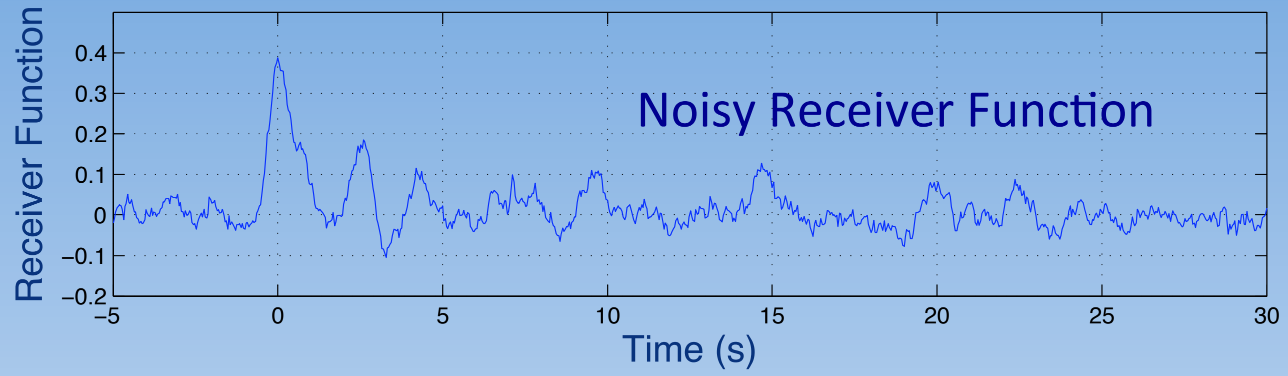
How do we parameterize C_D?

$$C_D = \sigma^2 \begin{bmatrix} 1 & r & r^2 & \dots & r^{N-1} \\ r & 1 & r & & r^{N-2} \\ r^2 & r & 1 & & r^{N-3} \\ & & & \vdots & \\ r^{N-1} & r^{N-2} & r^{N-3} & \dots & 1 \end{bmatrix}$$

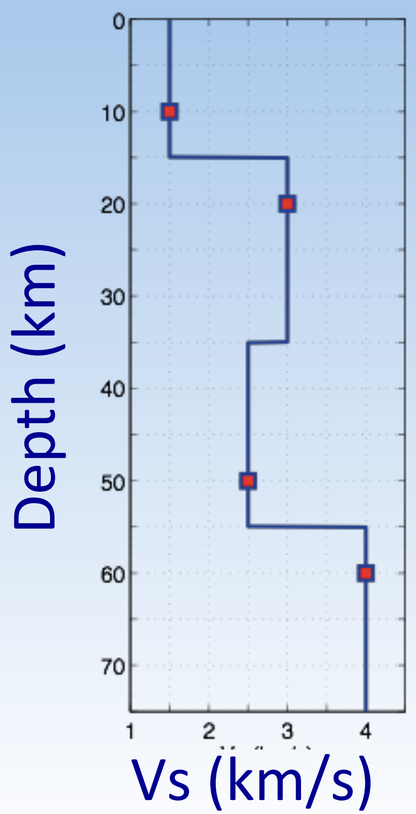
2 parameters :

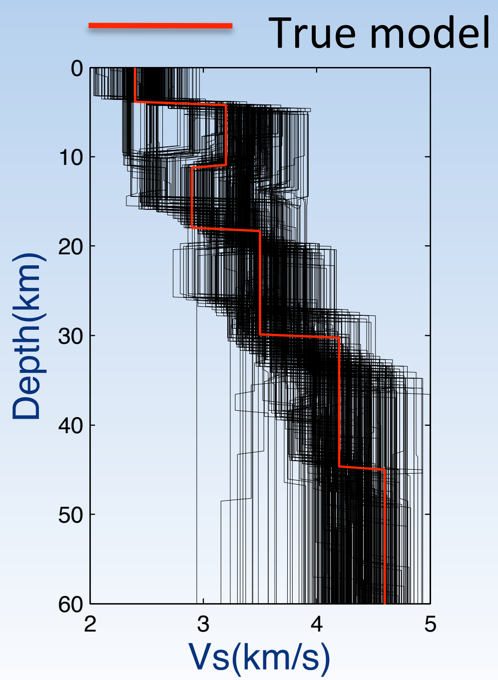
Magnitude of noise σ
Correlation of noise r

Synthetic experiment



Magnitude and correlation of noise are treated as unknowns

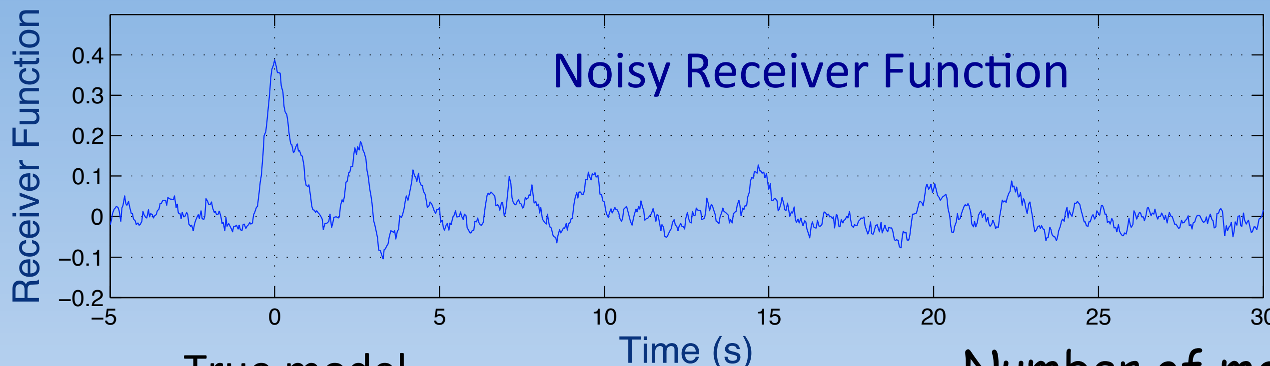




Solution is a large ensemble of models distributed according to the target distribution

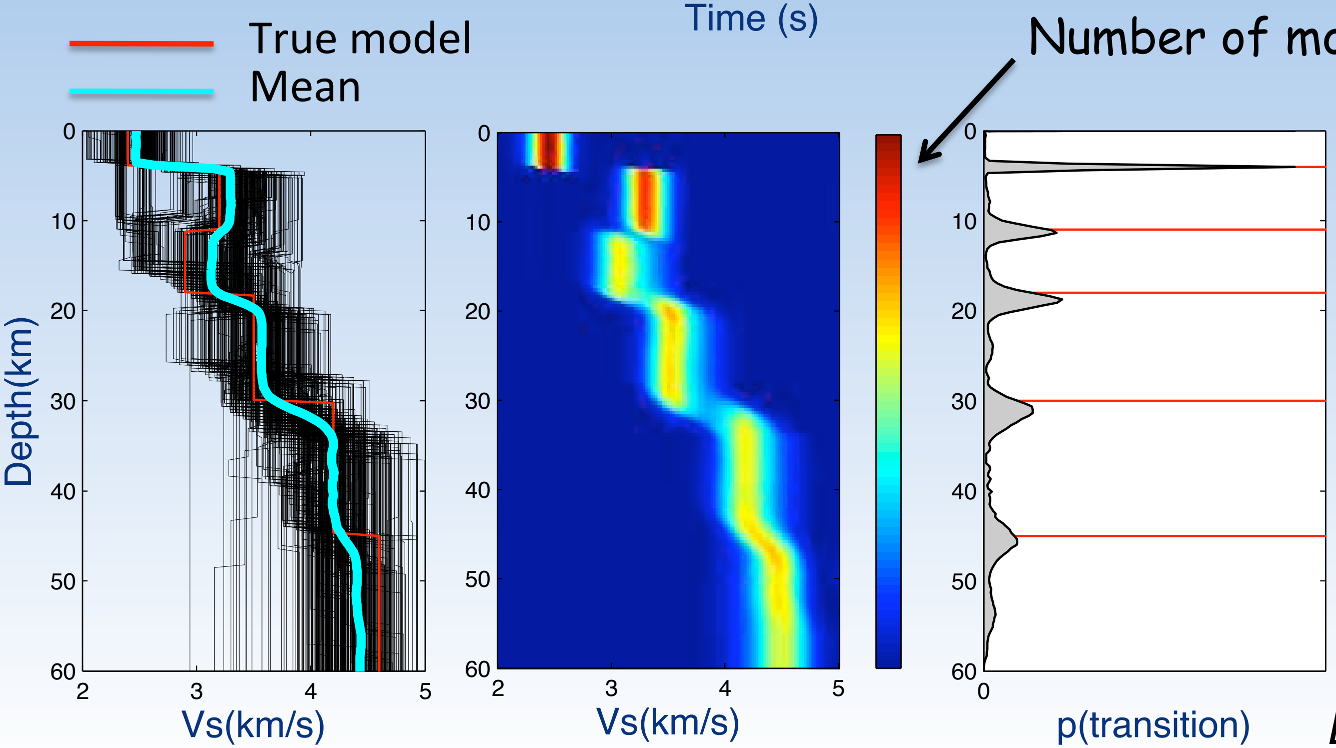
Bodin et al., JGR 2012

Synthetic experiment



Magnitude and correlation of noise are unknown

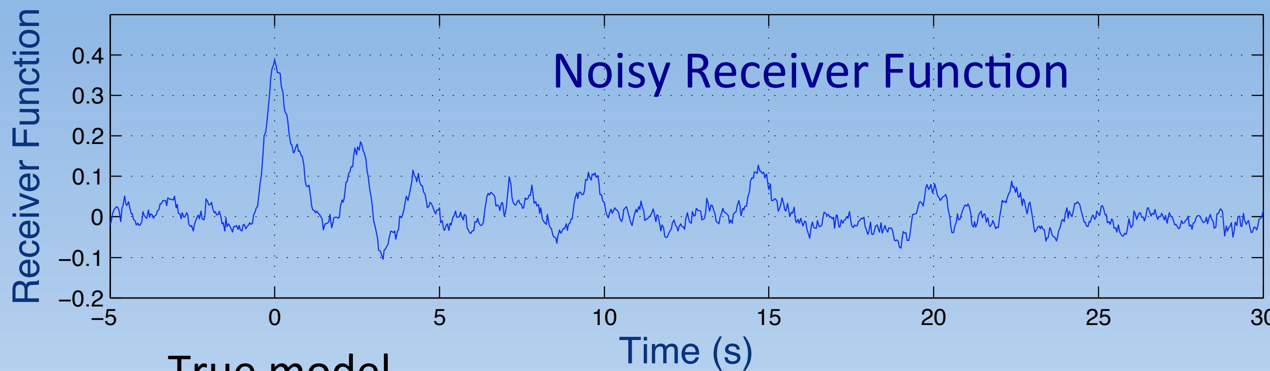
Number of models



Different ways to look at the solution

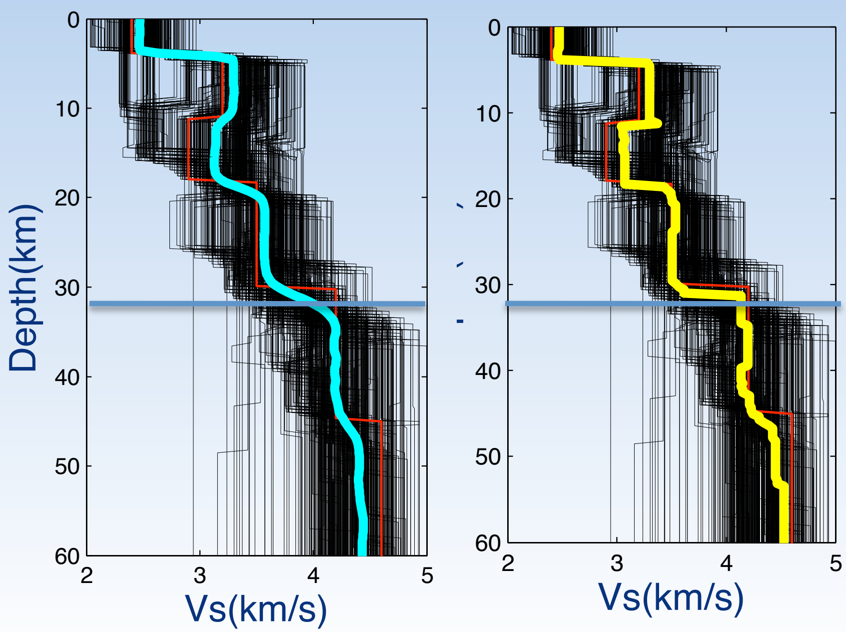
Bodin et al., JGR 2012

Synthetic experiment

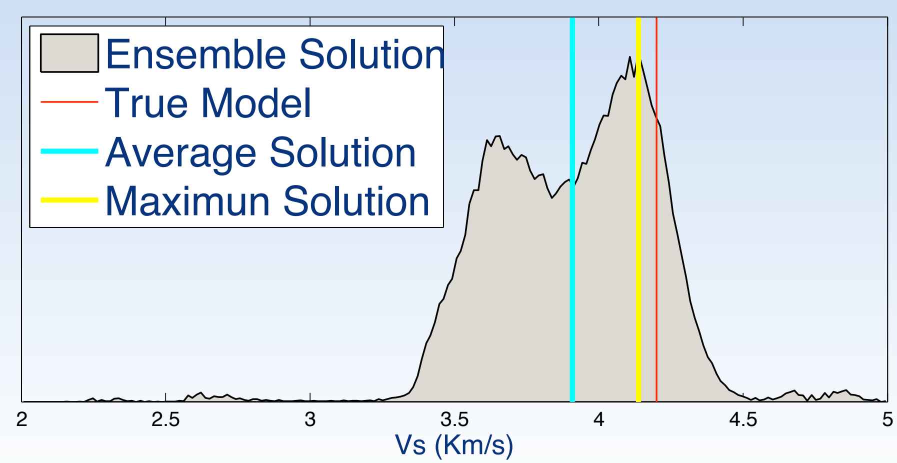


Magnitude and correlation of noise are unknown

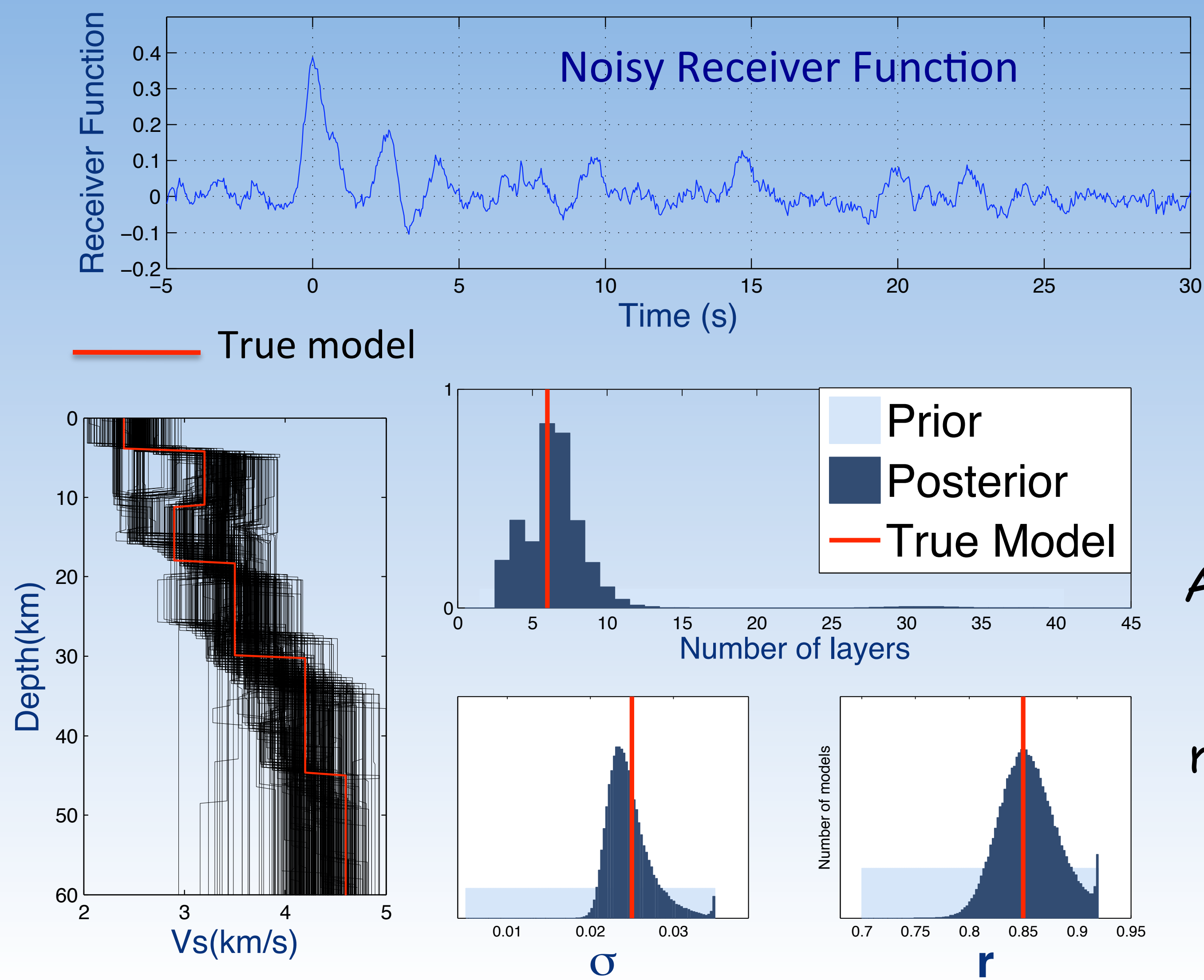
True model
Mean
Solution for maximum number of models



Probability distribution at 31 km



Synthetic experiment

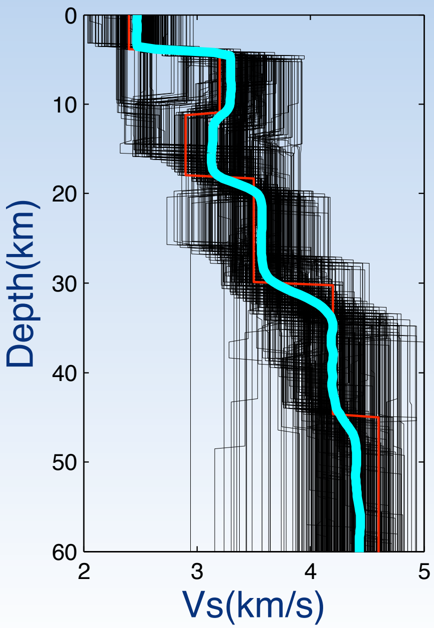


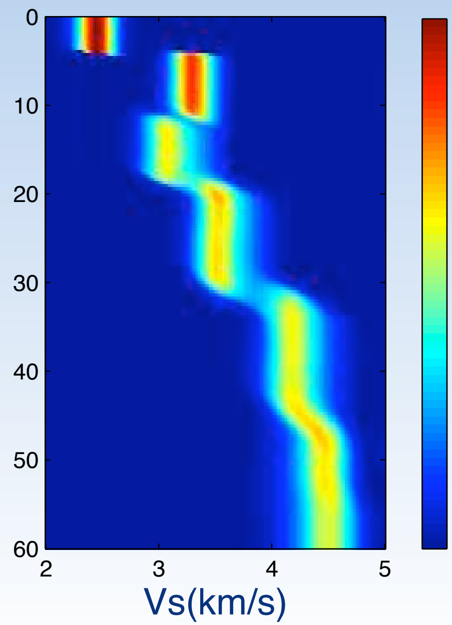
Algorithm is able to recover the complexity of the model and the level of data noise

Synthetic experiment

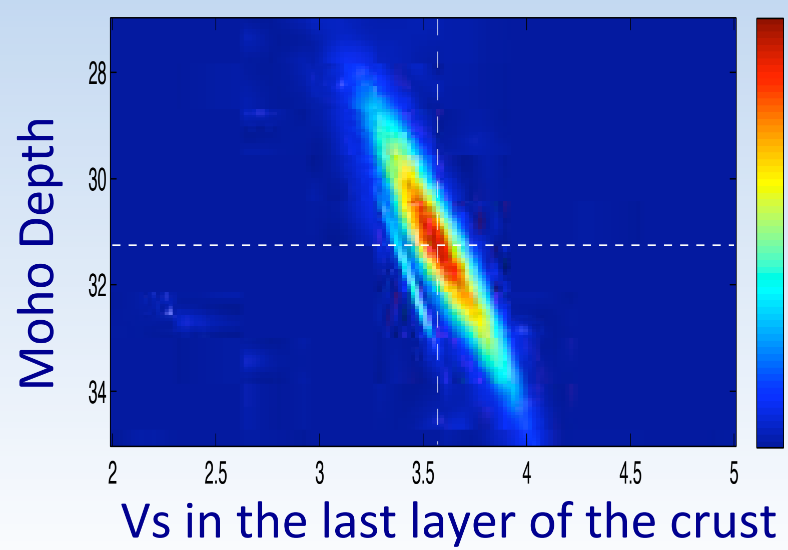
Magnitude and correlation of noise are unknown

True model

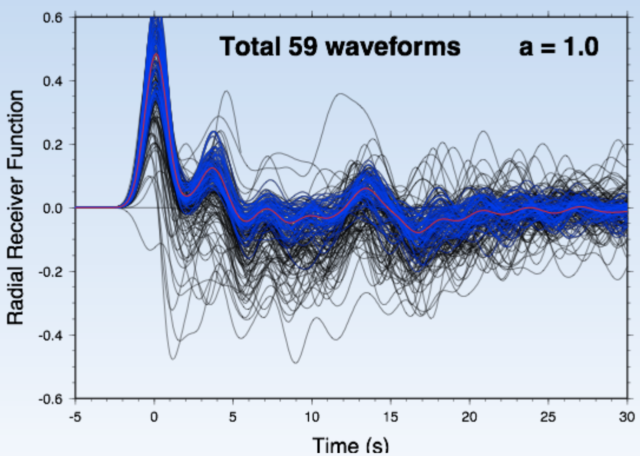


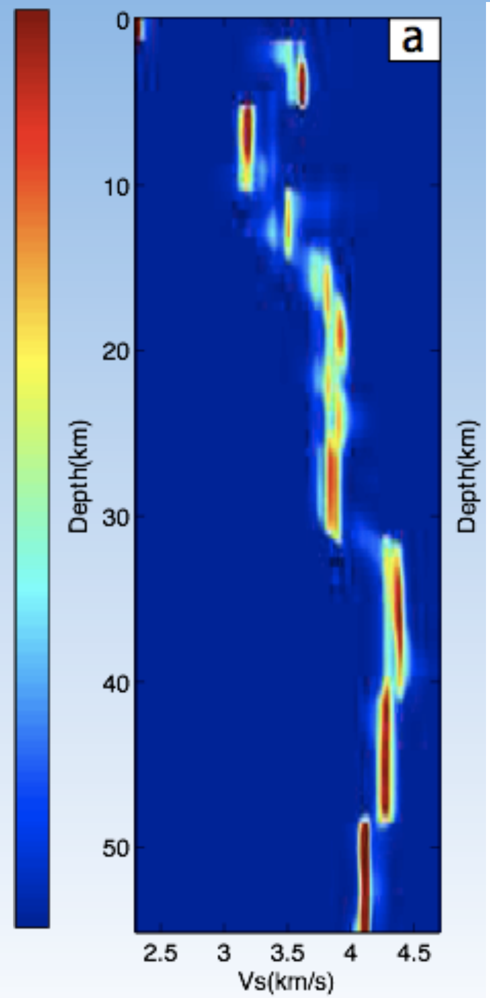


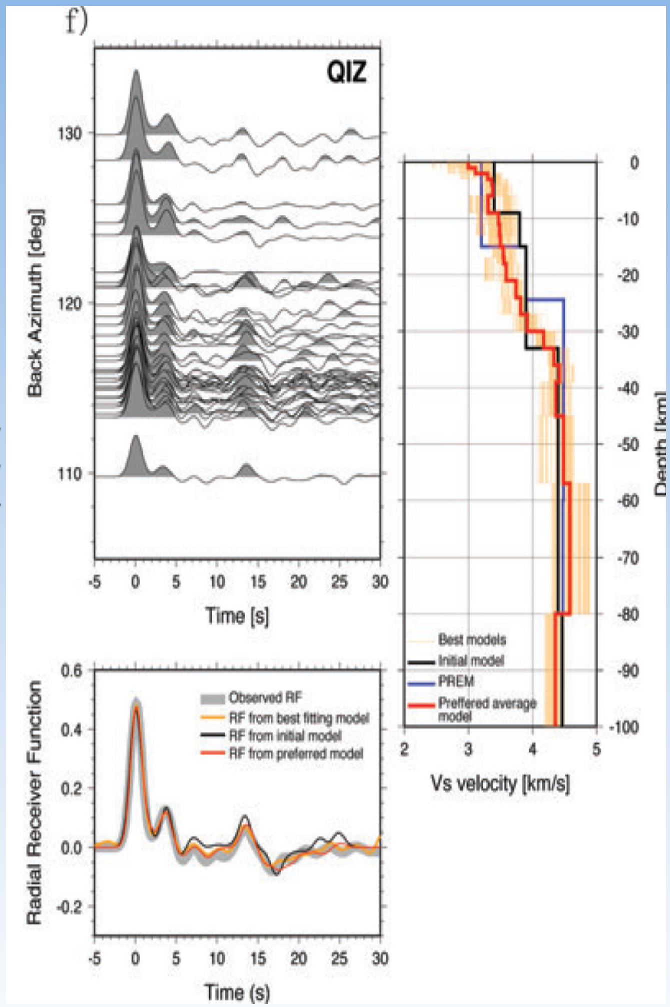
Trade-offs between parameters



Application to field data



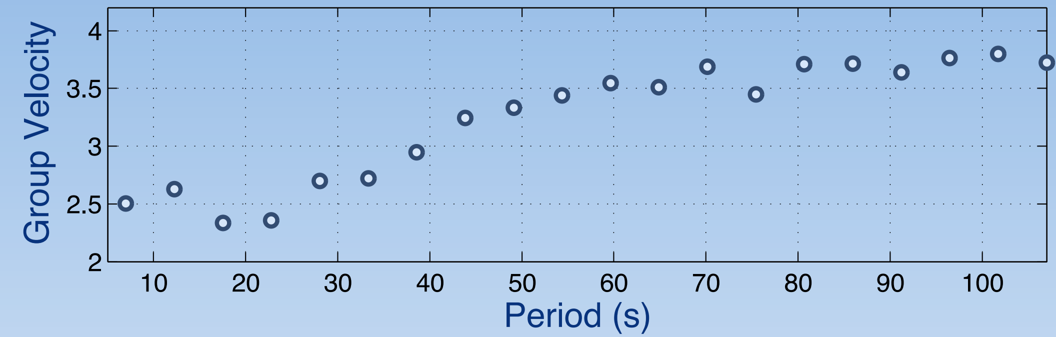




Bodin et al., JGR 2012

Application to Joint Inversion

Dispersion curve for Rayleigh waves



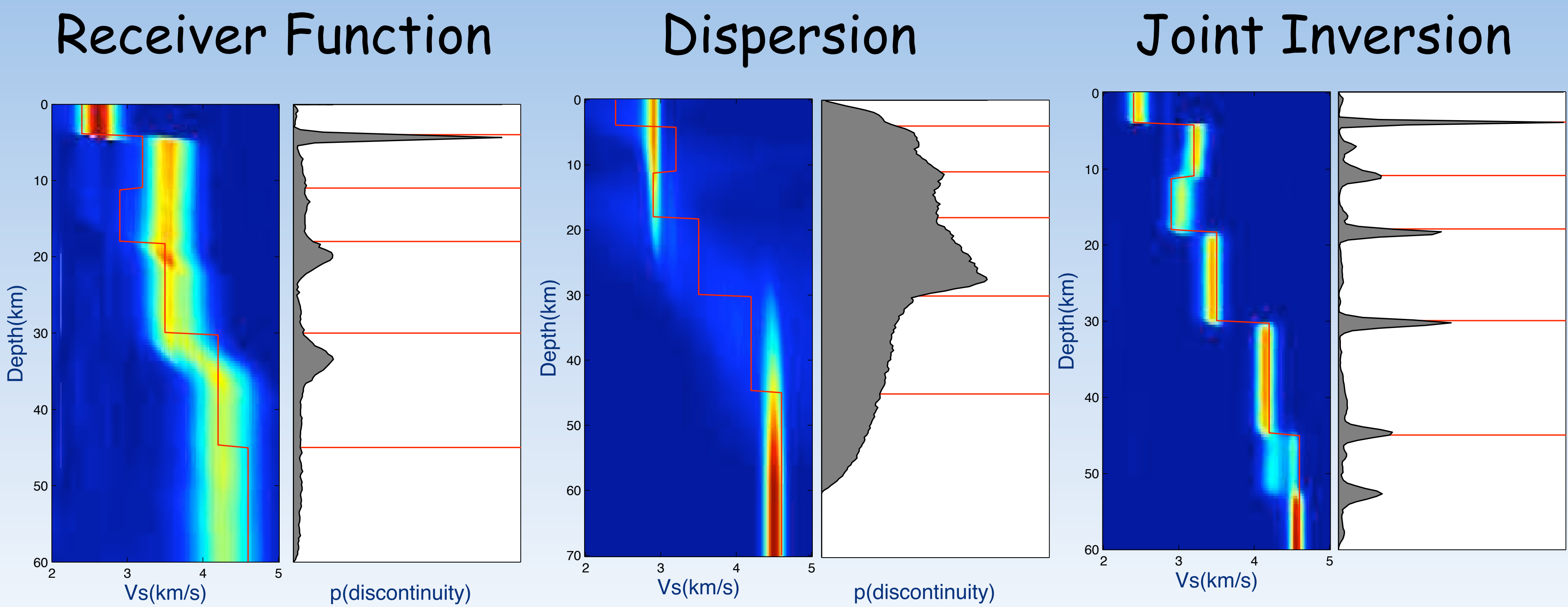
Receiver Function

$$d = d_{RF} + d_{dis}$$

$$C_D = \begin{bmatrix} C_D^{RF} & 0 \\ 0 & C_D^{dis} \end{bmatrix}$$

Algorithm naturally weights the information brought by each data type.

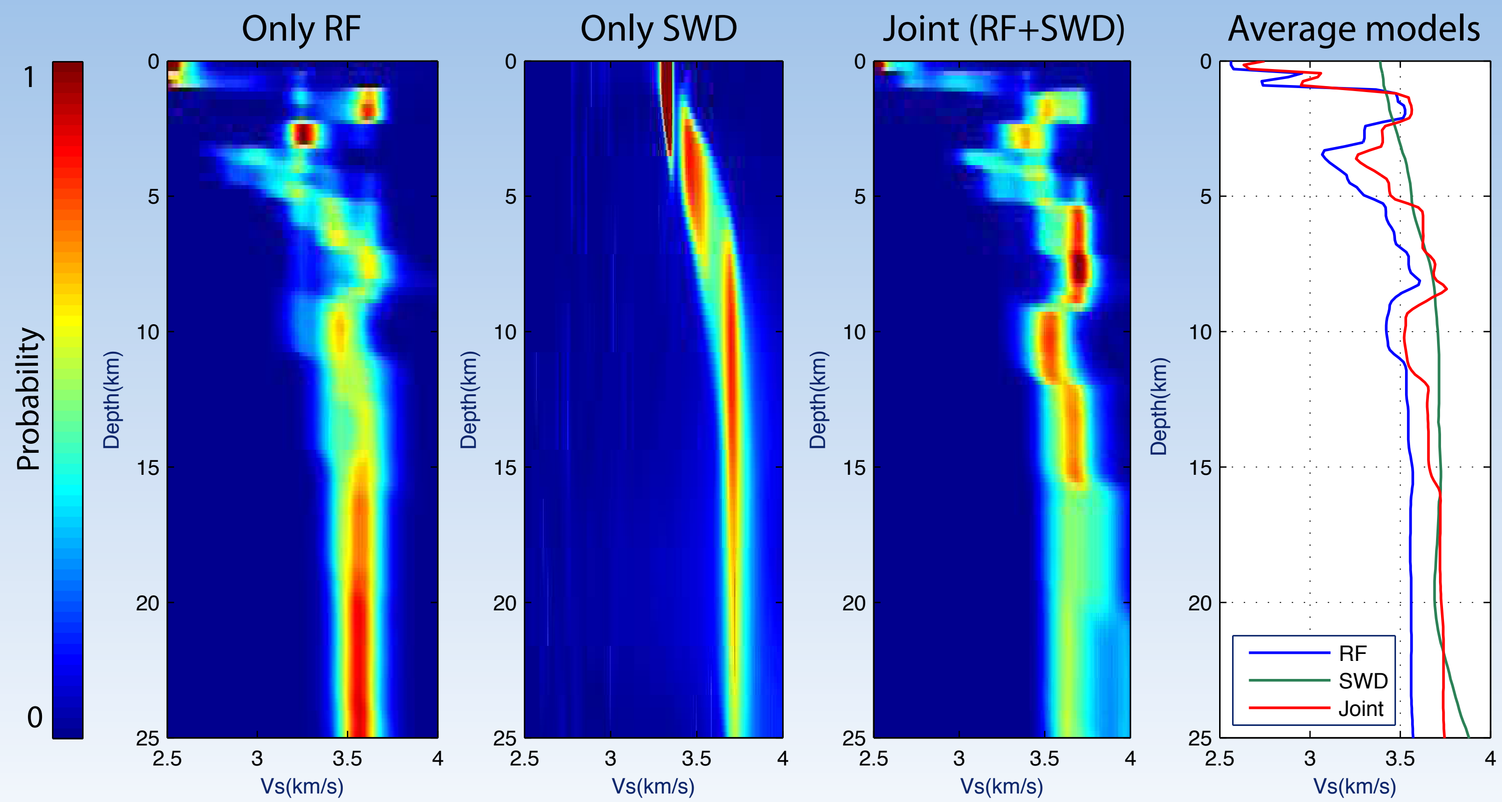
Application to Joint Inversion (synthetic test)



Bodin et al., JGR 2012

Application to Joint Inversion

WOMBAT data from SE Australia:
RFs + ambient noise dispersion



Bodin et al., JGR 2012

Future research

Modelling multiple geophysical datasets will be approached through the transdimensional hieararchical Bayesian framework, where the number of free parameters and the data noise will be treated as unknowns in the inversion.

Various simplifications that hinder the progress in crustal and lithospheric imaging using passive-source data and permanent/temporary seismic receivers will be gradually incorporated in the Bayesian inversion strategy – this includes, but is not limited to: anisotropy, dipping layers and 3D structure, noisy data, etc.

This is a general strategy that can be applied to other types of inverse problems in Earth Science and for imaging various parts of the Earth's crust.